

Generational Structure of Models with Dynamical Symmetry Breaking

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In models with dynamical electroweak symmetry breaking, this breaking is normally communicated to quarks and leptons by a set of vector bosons with masses generated via sequential breaking of a larger gauge symmetry. In reasonably ultraviolet-complete theories of this type, the number of stages of breaking of the larger gauge symmetry is usually equal to the observed number of quark and lepton generations, $N_{gen.} = 3$. Here we investigate the general question of how the construction and properties of these models depend on $N_{gen.}$, regarded as a variable. We build and analyze models with illustrative values of $N_{gen.}$ different from 3 (namely, $N_{gen.} = 1, 2, 4$) that exhibit the necessary sequential symmetry breaking down to a strongly coupled sector that dynamically breaks electroweak symmetry. Our results for variable $N_{gen.}$ show that one can robustly obtain, for this latter sector, a theory with a gauge coupling that is large but slowly running, controlled by an approximate infrared fixed point of the renormalization group. Owing to this, we find that for all of the values of $N_{gen.}$ considered, standard-model fermions of the highest generation have masses that can be comparable to the electroweak-symmetry breaking scale. We also study the interplay of multiple strongly coupled gauge symmetries in these models.

I. INTRODUCTION

Electroweak symmetry breaking (EWSB) plays a crucial role for observed particle interactions, but its origin remains an outstanding mystery. There are actually several aspects to this physics; in addition to a mechanism to explain how the W and Z bosons acquire masses, there is also the necessity to explain how the quarks and charged leptons gain masses, why they come in three generations, and why their masses exhibit the generational hierarchy that they do. Explaining the very small neutrino masses is yet another challenge. In an appealing class of theories with dynamical electroweak symmetry breaking, this breaking is produced by means of an asymptotically free, vectorial, gauge interaction based on an exact gauge symmetry, commonly called technicolor (TC), that becomes strongly coupled on the TeV scale, causing the formation of bilinear technifermion condensates [1]. In these theories, the EWSB is communicated to the standard-model (SM) fermions, which are technisinglets, via exchanges of massive gauge bosons associated with a higher symmetry, extended technicolor (ETC) [2]. Some early related works on gauge symmetry breaking include [3]; some recent reviews of TC/ETC theories include [4]–[7].

An important aspect of such theories is the pattern of sequential extended technicolor symmetry breaking to the residual technicolor symmetry, since this determines the hierarchical generational mass spectrum of the standard-model fermions. Early works tended to model ETC effects via (non-renormalizable) four-fermion operators connecting SM fermions and technifermions, with some assumed values for their coefficients. More complete studies took on the task of deriving these four-fermion operators by analyses of renormalizable, reasonably ultraviolet-complete, ETC theories. In particular, detailed studies were carried out for reasonably ultraviolet-complete ETC models containing an $SU(2)_{TC}$

technicolor gauge group, with ETC symmetry-breaking patterns giving rise to the observed three generations [8] of standard-model fermions [9] and to acceptably light neutrinos [10]–[14], [4]. As is evident from these models, there is a tight connection between the number of standard-model fermion generations and the sequential breaking of the ETC symmetry down to the technicolor subgroup.

The studies of reasonably ultraviolet-complete TC/ETC models naturally lead one to investigate a more general and abstract topic, namely the connection between the properties of the enveloping ETC theory and the number of standard-model fermion generations, $N_{gen.}$, when $N_{gen.}$ is taken as a variable rather than being fixed at its inferred physical value of 3 [8]. We address this question here, considering the hypothetical values $N_{gen.} = 1, 2$, and 4. The purpose of our analysis is not to try to produce a quasi-realistic ETC model, but instead to investigate how the value of $N_{gen.}$ influences the construction and properties of the model. There are several interesting questions that one can investigate in this context. In several previous detailed studies of reasonably ultraviolet-complete quasi-realistic TC/ETC models [9], [10]–[14], one relied upon an auxiliary strongly coupled gauge symmetry, called hypercolor (HC), to produce the requisite sequential ETC symmetry breaking. A natural question to ask is whether, for values of $N_{gen.}$ different from 3, in particular, for the apparently simpler cases $N_{gen.} = 1$ or $N_{gen.} = 2$, one might be able to construct a TC/ETC theory in which all of the ETC symmetry breaking could be accomplished by strong self-breaking of the ETC gauge symmetry, without the aid of this auxiliary strongly coupled HC gauge interaction. A second and related topic for investigation is the evolution of the ETC and HC interactions from high energy scales down to lower ones, through the various sequential breakings of the

high-scale ETC symmetry. This entails an examination of the effective field theories that are operative at the different scales, including their content of dynamical fermions and their plausible channels of condensation. One aspect of this is to check that the relevant strongly coupled gauge interactions do, indeed, plausibly produce the requisite bilinear fermion condensates for ETC symmetry breaking and, in the case of the residual TC theory, the technifermion condensates that cause dynamical electroweak symmetry breaking, rather than evolving into the infrared in a chirally symmetric manner, with unwanted non-Abelian Coulombic behavior. Yet another interesting question concerns the effect of changing the value of $N_{gen.}$ on the properties of the residual technicolor theory, and whether this TC theory can easily have a large but slowly running gauge coupling associated with an approximate infrared fixed point. A profound mystery pertaining to the observed quarks and leptons is why all of them except for one – the top quark – have masses that are considerably smaller than the electroweak symmetry-breaking scale. Thus, a final question concerns what generic predictions these models with other values of $N_{gen.}$ make about the standard-model quarks and leptons. Our illustrative models with variable $N_{gen.} \neq 3$ provide a useful theoretical laboratory in which to investigate these questions.

Although our main focus here is on the general field-theoretic question of the role of $N_{gen.}$ in TC/ETC model-building, we note that there is currently continuing interest in the possibility that there really are four (or more) generations of SM fermions. Reviews include Refs. [15], [20], and, up to 2009, [17], and some recent papers include [18]. Most of this work has been done within the context of a Higgs mechanism as the origin of electroweak symmetry breaking. It has been noted, in particular, that if the fermions of the fourth generation are sufficiently heavy, i.e., the corresponding Yukawa couplings are sufficiently large, then the Higgs interactions become nonperturbative and can produce fermion condensates. There are triviality upper limits on Yukawa couplings obtained from fully nonperturbative dynamical-fermion lattice simulations [19]. Current experimental lower limits on the masses of possible sequential fourth-generation quarks and leptons are given in [20] but depend on various assumptions such as the ordering of masses of the fourth-generation quarks and leptons and the values of relevant mixing angles.

Before proceeding, some remarks on the current status of (extended) technicolor models are in order. These theories are very ambitious, since they incorporate a dynamical origin not just for electroweak symmetry breaking, but also for fermion masses. This contrasts with the standard model, which obtains electroweak symmetry breaking by fiat, from the choice of the sign of the coefficient of the quadratic term in the Higgs potential, and accommodates, but does not explain, the observed quark and charged lepton masses by appropriate choices of Yukawa

couplings. (Here and below, by the term “standard model”, we implicitly mean an appropriate extension of the original standard model to account for nonzero neutrino masses.) Moreover, theories with dynamical EWSB are subject to a number of constraints from data on flavor-changing neutral-current processes, splitting of the m_t and m_b masses, precision electroweak measurements, limits on pseudo-Nambu-Goldstone bosons, etc. A fully realistic theory of this type would answer such longstanding questions as why mass ratios like m_e/m_μ have the values that they do. Given such ambitious goals, it is perhaps not surprising that no fully realistic TC/ETC model has yet been constructed.

However, there have been a number of important advances in this area. It was shown that technicolor theories can exhibit a large but slowly running (“walking”) coupling [21], as a consequence of an approximate infrared fixed point in the renormalization group equation for the TC gauge coupling [22]–[24]. Recently, there has been important progress in elucidating the properties of walking gauge theories by means of lattice simulations. For the case considered here, of (techni)fermions in the fundamental representation, these recent lattice studies include the works in Ref. [25]. There have also been studies of walking gauge theories with (techni)fermions in higher-dimensional representations (a few include [26, 27]; see also the review [7]). The walking behavior enables TC/ETC theories to generate sufficiently large fermion masses to match experiment with ETC mass scales that are large enough to avoid excessive flavor-changing neutral-current effects. Furthermore, this walking behavior may be able to reduce technicolor contributions to W and Z propagators to a level in agreement with experimental limits (e.g., [28]–[30], [6, 7], and references therein). One of the results obtained from a detailed study of a reasonably ultraviolet ETC theory was the demonstration [10, 12] that models of this type, in which the SM fermions transform as vectorial representations of the ETC gauge group, did not have as severe problems with flavor-changing neutral-current processes as had previously been thought on the basis of less ultraviolet-complete models. For example, one of the most severe constraints had been considered to arise from $K^0 - \bar{K}^0$ mixing. However, as was pointed out in [12], this would proceed via the $d\bar{s}$ in the K^0 producing a virtual V_2^1 ETC gauge boson (where the numbers are the gauged generational indices), but the $s\bar{d}$ in the final-state \bar{K}^0 can only be produced by a V_1^2 . Hence, the $K^0 - \bar{K}^0$ transition can only proceed via a nondiagonal ETC gauge boson mixing, $V_2^1 \rightarrow V_1^2$. Having an ultraviolet-complete ETC theory, one could calculate this mixing quantitatively; this was done in Refs. [10, 12], and it was found to suppress the $K^0 - \bar{K}^0$ transition strongly.

The central role that the number of standard-model fermion generations, $N_{gen.}$, plays in the structure of TC/ETC theories may be contrasted with the rather different role that it plays in the standard model, su-

persymmetric extensions thereof, and (supersymmetric) grand unified theories. In these three types of theories, at least when viewed as pointlike field theories without being derived from a string theory, one puts in the value of $N_{gen.}$ as copies of the fermion representations. The number $N_{gen.} = 3$, as such, does not play a direct role in the symmetry breaking of the grand unified symmetry or in the determination of the fermion masses. Besides the (extended) technicolor approach, only very few other approaches have attempted to predict $N_{gen.}$ from intrinsic properties of the model rather than inserting it by hand. One effort in this direction was based on composite models of SM fermions [31]. A particularly elegant approach is provided by string theory, in which $N_{gen.}$ is determined by the topology of the compactification manifold or orbifold M (and resultant number of zero modes of the Dirac operator), as given by $|\chi(M)|/2$, where χ is the Euler characteristic of M [32]. Here we focus on the more bottom-up approach provided by extended technicolor.

II. GENERAL THEORETICAL FRAMEWORK AND CALCULATIONAL METHODS

A. Gauge Group

We consider a $((3+1)$ -dimensional) gauge theory with the gauge group

$$G = \text{SU}(N_{ETC})_{ETC} \times \text{SU}(N_{HC})_{HC} \times G_{SM} , \quad (2.1)$$

where

$$G_{SM} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \quad (2.2)$$

is the standard-model gauge group and $\text{SU}(N_{ETC})_{ETC}$ is the extended technicolor gauge group, which dynamically breaks to the technicolor group $\text{SU}(N_{TC})_{TC}$ in a series of stages at successively lower and lower energy scales. In order to communicate the electroweak symmetry breaking to the standard-model fermions, the ETC group gauges the generational indices and combines them

with the technicolor indices, so that

$$N_{ETC} = N_{gen.} + N_{TC} . \quad (2.3)$$

As in Refs. [10, 12], we take $N_{TC} = 2$ because this minimizes technicolor corrections to W and Z propagators. As we will demonstrate, it also allows us to obtain walking behavior for the residual technicolor sector for each of the values of $N_{gen.}$ that we study, generalizing the previous success in obtaining walking behavior for $N_{gen.} = 3$. The extended technicolor sector is arranged to be an asymptotically free chiral gauge theory. In earlier ETC models [9], [10]-[4] with $N_{gen.} = 3$, in order to obtain the desired sequential breaking of the ETC gauge symmetry, one included another strongly coupled gauge interaction, called hypercolor (HC). The hypercolor gauge group was taken to be $\text{SU}(2)_{HC}$. There were several reasons for this choice, including minimality and the fact that $\text{SU}(2)$ has only (pseudo)real representations, so that there are no gauge anomalies; this gives one added flexibility in choosing the representations of hypercolored fermions. As noted above, one of the questions that we will examine here is whether for the lower values of $N_{gen.} = 1$ or 2, it might be possible to simplify the model by eliminating the hypercolor interaction, so that all of the ETC symmetry breaking is produced by ETC itself, as self-breaking. To anticipate our results, and as indicated in Eq. (2.1), the models that we have constructed with the requisite ETC breaking patterns still rely on hypercolor. We take the gauge symmetry (2.1) as our starting point but mention that there have also been studies of ideas for deriving $N_{gen.}$ from a higher unification of gauge symmetries in a TC/ETC context [33].

B. Fermion Content

The fermion content of each the models includes $N_{gen.}$ generations of standard-model quarks and leptons, arranged together with technifermions with the same SM quantum numbers in the following ETC multiplets:

$$Q_L : (N_{ETC}, 1, 3, 2)_{1/3, L} , \quad u_R : (N_{ETC}, 1, 3, 1)_{4/3, R} , \quad d_R : (N_{ETC}, 1, 3, 1)_{-2/3, R} , \quad (2.4)$$

and

$$L_L : (N_{ETC}, 1, 1, 2)_{-1, L} , \quad e_R : (N_{ETC}, 1, 1, 1)_{-2, R} . \quad (2.5)$$

Here the numbers in parentheses refer to the dimensions of the representations under $\text{SU}(N_{ETC})_{ETC} \times \text{SU}(2)_{HC} \times \text{SU}(3)_c \times \text{SU}(2)_L$ and the subscript gives the weak hypercharge, Y . We will also use the notation U , D , E , and

n for the technifermions with the indicated SM quantum numbers. Thus, for example, for the case $N_{gen.} = 3$,

$$e_R \equiv (e^1, e^2, e^3, e^4, e^5)_R \equiv (e, \mu, \tau, E^4, E^5)_R . \quad (2.6)$$

Note that the set (2.5) does not contain right-handed, electroweak-singlet neutrinos; these will arise as residual components of SM-singlet fermions that transform according to larger representations of the ETC gauge group. Refs. [12, 14] also investigated a different set of fermion representations with $Q_L : (5, 1, 3, 2)_{1/3, L}$, $u_R : (5, 1, 3, 1)_{4/3, R}$, $d_R : (5, 1, 3, 1)_{-2/3, R}$, $L_L : (5, 1, 1, 1)_{-1, L}$, and $e_R : (5, 1, 1, 1)_{-2, R}$, and containing a corresponding set of SM-singlets that rendered the theory anomaly-free. However, while that set of fermions produces a natural splitting in m_t and m_b (and m_c and m_s) without excessive violation of custodial symmetry, it leads to flavor-changing neutral current effects [12–14] that are too large. Other TC/ETC strategies for getting large splitting between m_t and m_b include the use of more than one ETC group (e.g., [34] and references therein) and the use of new gauge interaction(s) such as topcolor [35]. We do not pursue these here, although it could be of interest in future work to study such models with variable $N_{gen.}$.

Masses for quarks and leptons arise from diagrams in which these fermions emit virtual ETC gauge bosons, making transitions to virtual technifermions, and then reabsorb the ETC gauge bosons. Since the contributions of the corresponding Feynman integrals to these masses depend on the ETC gauge boson masses via their propagators, exchanges of the heaviest ETC gauge bosons, with masses of order the highest scale of ETC symmetry breaking, produce the smallest fermion masses, namely those of the first generation. For this reason, this highest ETC breaking scale is denoted Λ_1 . In quasi-realistic ETC models with $N_{gen.} = 3$, exchanges of ETC gauge bosons with masses of order the next lower ETC symmetry breaking scale play a dominant role in determining the masses of second-generation SM fermions, motivating the notation Λ_2 for this scale, and similarly for the lowest ETC scale, Λ_3 , and the third generation. These exchanges give rise to the diagonal elements of the respective 3×3 mass matrices of the quarks of charge $2/3$ and $-1/3$ and the charged leptons. Incorporating ETC gauge boson mixing is required to produce off-diagonal elements of these mass matrices, as studied in [12, 14]. The observed quark mixing matrix arises from the differences in the mixings in the charge $2/3$ and $-1/3$ quark sectors. Accounting for the very light neutrino masses requires an additional mechanism in which ETC gauge boson mixing yields Dirac neutrino mass terms connecting left- and right-handed neutrinos, and Majorana masses for right-handed (electroweak-singlet) neutrinos, leading to a low-scale seesaw [10–12]. In quasi-realistic $N_{gen.} = 3$ ETC models that are reasonably ultraviolet-complete (at least up to 10^4 TeV), such as those studied in Ref. [10, 12], one typically has $\Lambda_1 \sim 10^3$, TeV, $\Lambda_2 \sim 10^2$ TeV, and $\Lambda_3 \sim \text{few TeV}$. These values are sufficient to produce the requisite fermion masses and also to satisfy constraints from flavor-changing neutral-current processes.

C. Sequential ETC Symmetry Breaking

Generalizing the process of sequential ETC gauge symmetry breaking to the case of present interest, in which $N_{gen.}$ is a variable rather than being fixed at the physical value of 3, we require that the model have the property that the ETC symmetry breaking occurs in $N_{gen.}$ sequential stages,

$$\text{SU}(N_{ETC})_{ETC} \rightarrow \text{SU}(N_{ETC} - 1)_{ETC} \quad \text{at } \Lambda_1, \quad (2.7)$$

then, for $N_{gen.} \geq 2$,

$$\text{SU}(N_{ETC}-1)_{ETC} \rightarrow \text{SU}(N_{ETC}-2)_{ETC} \quad \text{at } \Lambda_2, \quad (2.8)$$

and so forth, down to

$$\text{SU}(3)_{ETC} \rightarrow \text{SU}(2)_{TC} \quad \text{at } \Lambda_{N_{gen.}}, \quad (2.9)$$

with

$$\Lambda_1 > \Lambda_2 > \dots > \Lambda_{N_{gen.}}. \quad (2.10)$$

For a given value of $N_{gen.}$, it is thus necessary to choose the fermion content so that the ETC theory undergoes this requisite sequential gauge symmetry breaking. If one is trying to construct a quasi-realistic TC/ETC model, then one has to do more than just obtaining the sequential ETC symmetry breaking at the scales in Eq. (2.10); one also has to ensure that the actual values of Λ_j with $j = 1, 2, 3$ yield acceptable SM fermion masses and mixings that are in reasonable agreement with experimental values. However, because of the strongly coupled nature of the physics, it is difficult to calculate the scales Λ_i precisely. To avoid electroweak symmetry breaking at too high a scale, the fermions that are responsible for this ETC symmetry breaking are taken to SM-singlets. Because the full theory is chiral, there are no fermion mass terms in the Lagrangian. The chiral fermions transform according to a set of representations $\{R_i\}$ of the ETC and HC groups. We denote the running gauge couplings of these groups at the reference energy scale $E = \mu$ as $g_{ETC} \equiv g_{ETC}(\mu)$ and $g_{HC} \equiv g_{HC}(\mu)$, and we define $\alpha_{ETC} = g_{ETC}^2/(4\pi)$ and $\alpha_{HC} = g_{HC}^2/(4\pi)$. (The implicit μ -dependence of these couplings will generally be suppressed in the notation.) The fermion content of the ETC and HC theories is chosen to incorporate the property that these two interactions are both asymptotically free. Hence, as the reference energy scale μ decreases from high values, α_{ETC} and α_{HC} increase. As the scale μ decreases through $\mu = \Lambda_1$, the HC and ETC interactions produce bilinear fermion condensates. Because of the chiral nature of the ETC fermion representations, these condensates generically break the ETC gauge symmetry. The fermions involved in the condensates gain dynamical masses of order Λ_1 and the gauge bosons corresponding to broken generators gain masses of order $g_{ETC} \Lambda_1$.

Following the principles of effective field theory, one analyzes the evolution of the theory to lower energy scales by integrating out these fields that gain masses at the

scale Λ_1 . One then proceeds to study the evolution of the low-energy effective field theory at energy scales $\mu < \Lambda_1$. For $N_{gen.} = 1$, this is the only stage of ETC symmetry breaking, while for $N_{gen.} \geq 2$, one analyzes the evolution of each of the $N_{gen.}$ effective field theories resulting from the corresponding stages of symmetry breaking and resultant acquisition of masses by the fermions involved in the condensates and by the gauge bosons corresponding to broken generators at each stage. The overarching ETC theory considered here is designed so that at each level of symmetry breaking, the descendent ETC and HC interactions remain asymptotically free, and hence the respective ETC and HC gauge couplings continue to grow, triggering the formation of the next set of condensates and the next stage of symmetry breaking [36].

D. Resultant Technicolor Theory

The result of the $N_{gen.}$ stages of ETC symmetry breaking is a theory invariant under the gauge group

$$\text{SU}(2)_{TC} \times \text{SU}(2)_{HC} \times G_{SM} . \quad (2.11)$$

Since the only HC-nonsinglet fermions are SM-singlets, their condensates are automatically invariant under G_{SM} . In contrast, the model is designed so that the technifermion condensates transform as operators with weak isospin $T = 1/2$ and weak hypercharge $|Y| = 1$, so these break the electroweak group $\text{SU}(2)_L \times \text{U}(1)_Y$ to electromagnetic $\text{U}(1)_{em}$. On the basis of vacuum alignment arguments, one infers that the technifermion condensates include

$$\langle \bar{F}_{i,L} F_R^i \rangle + h.c. , \quad (2.12)$$

where here F refers to U , D , and E and the sum over i is over $\text{SU}(2)_{TC}$ gauge indices. For technineutrinos, the left- and right-handed components in the models of Refs. [10, 12] and also in most of our present models transform according to conjugate representations of $\text{SU}(2)_{TC}$, and hence the condensates for these are $\langle \epsilon^{ij} \bar{n}_{i,L} n_{j,R} \rangle + h.c.$, leading to the effective mass terms

$$- \Lambda_{TC} \sum_i \epsilon^{ij} \bar{n}_{i,L} n_{j,R} + h.c. \quad (2.13)$$

(In the specific models below, the $n_{j,R}$ will arise from various ETC representations with different labels, such as $\psi_{j,R}$; we use the $n_{j,R}$ notation here to indicate the technineutrino components.)

The W and Z bosons gain masses given by

$$m_W^2 = \frac{g^2 F_{TC}^2 N_D}{4} \quad (2.14)$$

and

$$m_Z^2 = \frac{(g^2 + g'^2) F_{TC}^2 N_D}{4} , \quad (2.15)$$

where g and g' are the $\text{SU}(2)_L$ and $\text{U}(1)_Y$ gauge couplings and N_D denotes the number of $\text{SU}(2)_{TC}$ technidoublets,

$$N_D = N_c + 1 = 4 , \quad (2.16)$$

and F_{TC} is the TC analogue of f_π . To fit experiment, $F_{TC} = 250$ GeV.

E. Gauge Coupling Evolution and Criteria for Fermion Condensation

The evolution of the various gauge couplings is determined by the respective beta functions. For a given gauge group G with gauge coupling g_G , the beta function is $\beta = dg_G/dt$, where $dt = d \ln \mu$. For our analysis, the beta functions for the ETC and HC groups will be of particular interest, since these are the ones that are relevant for the sequential breaking of the ETC symmetry down to technicolor. We have

$$\beta(g_G) = -g_G \sum_{\ell=1}^{\infty} b_{G,\ell} \left(\frac{g_G^2}{16\pi^2} \right)^\ell \quad (2.17)$$

where $b_{G,\ell}$ arises at ℓ -loop order in perturbation theory, and we will focus on the first two coefficients, $b_{G,1}$ and $b_{G,2}$, since they are the only scheme-independent ones. Equivalently, with $\alpha_G = g_G^2/(4\pi)$,

$$\frac{d\alpha_G}{dt} = -\frac{\alpha_G^2}{2\pi} \left[b_{G,1} + \frac{b_{G,2} \alpha_G}{4\pi} + O(\alpha_G^2) \right] . \quad (2.18)$$

We will apply these results for G equal to the ETC, TC, and HC groups, respectively. To avoid cumbersome notation, henceforth we will suppress the subscript G where no confusion will result. Since the ETC and HC interactions are asymptotically free, it follows that in each case, $b_1 > 0$. If there are sufficiently many fermions that are nonsinglets under a given interaction, b_2 reverses sign from positive to negative, and, in this case, the perturbative beta function has a infrared zero away from the origin, given, to this order, by

$$\alpha_{IR} = -\frac{4\pi b_1}{b_2} . \quad (2.19)$$

This perturbative IR zero is important as a natural origin for walking technicolor. This will be especially important for the one-family technicolor sector incorporated in our models, since an analysis using the two-loop beta function and the Dyson-Schwinger equation for the technifermion propagator suggests that this theory exhibits an approximate infrared zero given by Eq. (2.19) and resultant walking behavior [22, 23]. (We note that a beta function may also exhibit a nonperturbative infrared zero away from the origin [37–39].)

Let us, then, consider a possible channel for chiral fermions, transforming as representations R_1 and R_2 under a given gauge group, to form a bilinear condensate transforming as $R_{cond.}$:

$$R_1 \times R_2 \rightarrow R_{cond.} . \quad (2.20)$$

An approximate measure of the attractiveness of this condensation channel is

$$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.}) , \quad (2.21)$$

where $C_2(R)$ is the quadratic Casimir invariant for the representation R [40]. A solution of the Dyson-Schwinger equation for a fermion propagator with zero input mass, in the approximation of single gauge boson exchange, yields a solution with a nonzero, dynamically generated mass if the gauge coupling exceeds the critical value α_{cr} given by

$$\frac{3 \Delta C_2 \alpha_{cr}}{2\pi} = 1 . \quad (2.22)$$

Corrections to this estimate have been studied in Ref. [41], but it will be sufficient for our purposes here. Since the dynamically generated mass multiplies the corresponding bilinear operator for this fermion in the effective Lagrangian, this is equivalent to the formation of a condensate for this operator. Some general formulas that are used in our calculations of the beta functions for the models are given in the appendix.

We comment on some other necessary conditions that an acceptable ETC model should meet. First, a model needs to satisfy the condition that the dynamical gauge symmetry breaking occurs at the highest scale in such a manner as not to break electroweak symmetry. This is not guaranteed, since, given that the SM-nonsinglet fermions (and technifermions) transform as specified in Eqs. (2.4) and (2.5), the theory contains the highly attractive possible condensation channel

$$N_{ETC} \times \bar{N}_{ETC} \rightarrow 1 , \text{ with } \Delta C_2 = \frac{N_{ETC}^2 - 1}{N_{ETC}} \quad (2.23)$$

involving these SM-nonsinglet fermions. This condensation channel must be avoided in the ETC theory for a number of reasons: (i) it would break electroweak symmetry at too high a scale; (ii) it would not break the ETC gauge symmetry, and hence (iii) it would not separate the usual SM fermions from the technifermions, and could produce a model in which all of these fermions are confined by the ETC interaction. In quasi-realistic models (which have $N_{gen.} = 3$) such as those of Refs. [4, 9, 10, 12], one avoids the occurrence of the unwanted condensation in the channel (2.23) by a hybrid mechanism which makes use of the fact that the model contains SM-singlet chiral fermions transforming as higher-dimensional representations of the ETC gauge group, and these can condense in a first stage of ETC symmetry breaking. For the second and third stages of ETC symmetry breaking, the models of Refs. [4, 9, 10, 12] rely on the effects of the auxiliary strongly coupled hypercolor gauge interaction. Another comment pertains to the sequential breaking of the ETC gauge symmetry. The model should be constructed in a manner that although this symmetry is chiral, the residual TC gauge symmetry

is vectorial and unbroken, so that it confines and produces the necessary bilinear technifermion condensates. The one-family technicolor sectors in the models that we consider here satisfy this condition.

III. A MODEL WITH $N_{gen.} = 1$

A. Field Content

We first study an ETC model that contains a single generation of standard-model fermions, $N_{gen.} = 1$. From Eq. (2.3), with the choice $N_{TC} = 2$, it follows that $N_{ETC} = 3$, so that the ETC gauge group is $SU(3)_{ETC}$. One constructs the ETC model to have a single stage of gauge symmetry breaking, viz.,

$$SU(3)_{ETC} \rightarrow SU(2)_{TC} . \quad (3.1)$$

The fermions which are nonsinglets under the SM gauge group are as given in Eqs. (2.4) and (2.5) with $N_{ETC} = 2$. We take the SM-singlet fermions to be

$$\psi_{i,R} : (\bar{3}, 1, 1, 1)_{0,R} \quad (3.2)$$

$$\chi_R^{i,\alpha} : (3, 2, 1, 1)_{0,R} \quad (3.3)$$

and

$$\omega_{\alpha,R} : (1, 2, 1, 1)_{0,R} , \quad (3.4)$$

where the numbers refer to the representations of $SU(3)_{ETC} \times SU(2)_{HC} \times SU(3)_c \times SU(2)_L$ and the subscripts denote the weak hypercharge, Y . This is an anomaly-free chiral gauge theory. Here we define the $SU(3)_{ETC}$ index $i = 1$ to refer to the single SM generation, while the indices $i = 2, 3$ are $SU(2)_{TC}$ indices. Note that since the total number of chiral fermions transforming according to the fundamental representation of $SU(2)_{HC}$ is even (equal to four), the theory is free of a global Witten anomaly associated with the homotopy group $\pi_4(SU(2)) = \mathbb{Z}_2$.

B. Condensation at Λ_1 Breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$

We proceed to study the evolution of this theory from high energy scales. The $SU(3)_{ETC}$ gauge interaction is asymptotically free, and the first two coefficients of the ETC beta function are $b_1 = 5$ and $b_2 = -12$. By Eq. (2.19), the perturbative two-loop beta function for the $SU(3)_{ETC}$ theory thus has a zero at $\alpha_{ETC,IR} = 5\pi/3$. Obviously, this prediction has considerable theoretical uncertainty because of the large value of the coupling.

The $SU(2)_{HC}$ sector is asymptotically free, with fermion content consisting of four chiral fermions, or equivalently, two Dirac fermions, transforming according as hypercolor doublets. From Eqs. (9.6) and (9.7)

with $N_{f,1/2} = 2N_{f,D,1/2} = 4$, it follows that the first two coefficients of the $SU(2)_{HC}$ beta function are $b_1 = 6$ and $b_2 = 29$. Since b_2 has the same sign as b_1 , this perturbative HC beta function does not have a zero away from the origin. The value $N_{f,D,1/2} = 2$ is well below the critical value, $N_{f,cr} \simeq 8$, where, according to the analysis of the Dyson-Schwinger equation for the fermion propagator (discussed further in the appendix), the theory would go over from one with confinement and spontaneous chiral symmetry breaking ($S\chi SB$) to a chirally symmetric one, which is often plausibly inferred to be a non-Abelian Coulomb phase. This property is important for our use of the HC interaction in this model, since it implies that as the energy scale μ decreases, the HC interaction will produce condensates of HC-nonsinglet fermions.

Since this theory has two asymptotically free gauge interactions, $SU(3)_{ETC}$ and $SU(2)_{HC}$, the properties of the theory depend on the relative sizes of the running couplings α_{ETC} and α_{HC} at a given reference energy scale μ . In order to obtain the desired pattern of symmetry breaking, we choose the initial value of the HC coupling at a high scale to be sufficiently large that the HC interaction plays the dominant role in breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$. This strategy is used because the ETC interaction, by itself, would favor condensation in the undesired channel (2.23), viz., in this case $3 \times \bar{3} \rightarrow 1$, involving the left and right-handed SM-nonsinglet fermions and technifermions. This unwanted condensation is avoided by making the HC interaction strong enough to play the controlling role in determining the formation of fermion condensates.

Thus, as the energy scale μ decreases from high values through a value that we shall denote as Λ_1 , the HC and ETC gauge couplings grows sufficiently large that together they produce a bilinear fermion condensate. Given the dominant role of the HC interaction, the most attractive channel involves the condensation of the $\chi_R^{i,\alpha}$. This channel is

$$(3, 2, 1, 1)_0 \times (3, 2, 1, 1)_0 \rightarrow (\bar{3}, 1, 1, 1)_0 \quad (3.5)$$

with $\Delta C_2 = 4/3$ for $SU(3)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon_{\alpha\beta} \epsilon_{ijk} \chi_R^{j\alpha} {}^T C \chi_R^{k\beta} \rangle, \quad (3.6)$$

where ϵ_{ijk} and $\epsilon_{\alpha\beta}$ are the totally antisymmetric tensor densities for $SU(3)_{ETC}$ and $SU(2)_{HC}$, respectively. (Condensation of the hermitian conjugate operator is implicitly understood to occur here and below.) As noted, the HC attraction plays a crucial role here, since as far as the ETC interaction itself is concerned, the channel (3.5) is less attractive than the channel $3 \times \bar{3} \rightarrow 1$, with $\Delta C_2 = 2C_2(\square) = 8/3$, involving the left and right chiral components of the SM fermions, which are HC-singlets. The condensation (3.5) breaks $SU(3)_{ETC}$ to $SU(2)_{TC}$ and is invariant under $SU(2)_{HC}$. With no loss of generality, we choose the uncontracted $SU(3)_{ETC}$ index to be $i = 1$, so that, carrying out the sum over repeated

$SU(3)_{ETC}$ indices in Eq. (3.6), one has, for the actual condensate,

$$2 \langle \epsilon_{\alpha\beta} \chi_R^{2,\alpha} {}^T C \chi_R^{3\beta} \rangle. \quad (3.7)$$

This condensate involves only SM-singlet fermions. With this symmetry breaking, the $i = 1$ components of the various fermion multiplets with nonsinglet SM quantum numbers split off from the other two components to become the first generation of SM fermions, while the remaining components, with $i = 2, 3$, are $SU(2)_{TC}$ technifermions. The fermions $\chi_R^{j\alpha} {}^T$ with $j = 2, 3$ involved in the condensate gain dynamical masses of order Λ_1 , and the five gauge bosons in the coset space $SU(3)_{ETC}/SU(2)_{TC}$ gain masses of order $g_{ETC} \Lambda_1 \simeq \Lambda_1$.

C. Condensation at $\Lambda'_1 < \Lambda_1$

The effective theory operative at energy scales just below Λ_1 is invariant under the strongly coupled gauge symmetries $SU(2)_{TC} \times SU(2)_{HC}$. Given that the HC interaction plays a dominant role in the formation of condensates, another one that forms in this theory at a scale that we shall denote Λ'_1 , slightly below Λ_1 , is driven by the HC interaction alone, without the help of $SU(2)_{TC}$. This involves the remaining component of the original $\chi_R^{i,\alpha}$ field, namely, $\chi_R^{1,\alpha}$, together with $\omega_{\alpha,R}$. These plausibly condense via the channel

$$(1, 2, 1, 1)_0 \times (1, 2, 1, 1)_0 \rightarrow (1, 1, 1, 1)_0, \quad (3.8)$$

where here the first number in the parentheses refers to the dimensionality of the representations of these fields under $SU(2)_{TC}$. The reason that this occurs at a scale somewhat below Λ_1 is that the channel (3.8) has the same measure of attractiveness as regards the HC interaction as the most attractive channel (3.5), namely, $\Delta C_2 = 3/2$, but does not receive any additional attraction from the TC interaction. The associated condensate is

$$\langle \chi_R^{1,\alpha} {}^T C \omega_{\alpha,R} \rangle. \quad (3.9)$$

This respects the same residual symmetry group as the condensate (3.7). As a result of the formation of this condensate, the $\chi_R^{1,\alpha}$ and $\omega_{\alpha,R}$ fermions gain dynamical masses of order Λ'_1 . In QCD the ratio of the lowest-lying ($J^{PC} = 0^{++}$) glueball mass to the QCD scale $\Lambda_{QCD} \simeq 250$ MeV is about $\kappa_g \simeq 7$. The present model (and the models with higher values of $N_{gen.}$) would also contain $SU(2)_{HC}$ -singlet bound states of hypercolor gluons at scales of order $\kappa_g \Lambda_1$ and $SU(2)_{TC}$ -singlet bound states of techniguons at scales of order $\kappa_g \Lambda_{TC}$. The actual mass eigenstates would be comprised of linear combinations of purely gluonic and fermion-antifermion states.

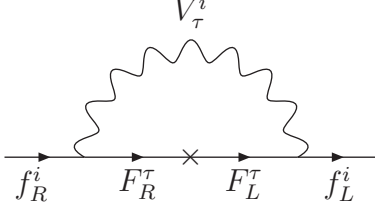


FIG. 1: Graph generating a SM fermion mass term, $m_{f^i} \bar{f}_{i,L} f_R^i$, diagonal in generational indices i . Here f stands for a SM quark or charged lepton, F for the corresponding technifermion, and τ for an $SU(2)_{TC}$ index.

D. Theory at Energy Scales Below Λ'_1 and SM Fermion Mass Generation

The masses of quarks and charged leptons arise from the one-loop diagram shown in Fig. 1. Here and below, it is understood that higher-loop diagrams also make important contributions because of the strong technicolor dynamics. The resultant value of the fermion masses for this $N_{gen.} = 1$ case is

$$m_f \simeq \kappa \eta \frac{\Lambda_{TC}^3}{\Lambda_1^2}, \quad (3.10)$$

where κ is a numerical factor of $O(10)$ (computed in Ref. [12]), and η is the renormalization factor, reflecting the shift in the running mass between the scales Λ_{TC} and Λ_1 :

$$\eta = \exp \left[\int_{\Lambda_{TC}}^{\Lambda_1} \frac{d\mu}{\mu} \gamma(\alpha(\mu)) \right], \quad (3.11)$$

where γ is the anomalous dimension for the technifermion bilinear and α refers to the TC coupling, as inherited from the enveloping ETC theory. With $N_{f,cr} \simeq 8$, as indicated by the beta function and Dyson-Schwinger analysis [23], the TC theory exhibits the slowest evolution of the coupling between Λ_{TC} and Λ'_1 , where there are 8 Dirac technifermions active. Since the anomalous dimension for the technifermion bilinear is generically $\gamma \simeq O(1)$ near the approximate IR fixed point of the renormalization group equation for the TC interaction, it follows that $\eta \sim \Lambda'_1/\Lambda_{TC}$, so that

$$m_f \simeq \kappa \frac{\Lambda'_1 \Lambda_{TC}^2}{\Lambda_1^2} \quad \text{for walking up to } \Lambda'_1. \quad (3.12)$$

In addition, owing to the presence of the approximate IR fixed point, the TC theory presumably also exhibits at least some walking behavior in the higher-lying interval between Λ'_1 and Λ_1 , where there are 9 Dirac technifermions active (including the $\psi_{j,R}$ with $j = 2, 3$). To the extent that the theory has walking behavior all the way up to Λ_1 with an associated large anomalous dimension for the technifermion bilinear, the renormalization

factor would be larger, viz., $\eta \sim \Lambda_1/\Lambda_{TC}$, and hence

$$m_f \simeq \kappa \frac{\Lambda_{TC}^2}{\Lambda_1} \quad \text{for walking up to } \Lambda_1. \quad (3.13)$$

In this case, with $\kappa \sim O(10)$, the model has the interesting feature that the fermion masses are comparable to the electroweak symmetry-breaking scale, given by Λ_{TC} . Even in the case where the TC theory exhibits walking only up to Λ'_1 , the fermion masses are considerably larger than the value $\kappa \Lambda_{TC}^3/\Lambda_1^2$ that they would have had in the absence of walking, since $\Lambda'_1/\Lambda_{TC} > 1$. The ratio Λ'_1/Λ_1 depends on the relative strength of the HC and ETC interactions, as measured by the ratio of the running couplings squared, α_{HC}/α_{ETC} . The larger this ratio is, the smaller is the ratio Λ'_1/Λ_1 . We find that the model does not have enough structure to lead to a low-scale seesaw mechanism for small, nonzero neutrino masses, as presented in Ref. [10] for a $N_{gen.} = 3$ model. Further ingredients would be necessary to produce such a seesaw.

At first sight, this model would appear to predict that for the fermions of the single generation, the masses of the charge 2/3 quark, the charge $-1/3$ quark, and the charged lepton are all equal. However, here the walking behavior of the technicolor theory can play yet another important role. Although the SM gauge interactions are small at the scale Λ_{TC} where the electroweak symmetry breaking occurs, such small perturbations would have a magnified effect on fermion masses in a theory with walking behavior. In turn, this has the potential to explain the relative sizes of the up-type, down-type, and charged lepton masses. The fact that the $SU(3)_c$ interaction contributes an attractive force that aids in the formation of the bilinear techniquark condensates $\langle \bar{U}_L U_R \rangle + h.c.$ and $\langle \bar{D}_L D_R \rangle + h.c.$ would mean that these condensates would naturally form at a somewhat higher scale than the technilepton condensate, and hence the dynamically generated mass for the techniquarks would be somewhat larger than that for the technileptons. This would provide a natural explanation for why the quarks of a given generation (a single generation here and multiple generations in other cases) have larger masses than the charged leptons. Moreover, the $U(1)_Y$ interaction is attractive for the $\langle \bar{U}_L U_R \rangle + h.c.$ condensate with $Y_{\bar{U}_L} Y_{U_R} = -4/9$, but repulsive for the $\langle \bar{D}_L D_R \rangle + h.c.$ condensate, with $Y_{\bar{D}_L} Y_{D_R} = 2/9$, so that the former condensate might be somewhat larger than the latter, and similarly for the corresponding dynamical techniquark masses. In this model, this would imply that the charge 2/3 quark is heavier than the charge $-1/3$ quark. (For leptons, the product $Y_{\bar{L}_L} Y_{e_R} = -2$ is also attractive.) However, one must remark that in a quasi-realistic model, the splitting of the dynamical masses of the charge 2/3 and charge $-1/3$ techniquarks could produce an excessively large violation of custodial symmetry. The explanation for the fact that $m_u < m_d$ for the first generation requires consideration of off-diagonal elements in the up and down-type quark mass matrices.

E. A Variant of the $N_{gen.} = 1$ Model

It is useful to discuss a variant of this model which provides an illustration of another problem of which one must be aware in TC/ETC model-building. This is the possibility that even if there are sufficiently few technifermions and their representations are sufficiently small that the technicolor interaction is asymptotically free, it may still happen that the technicolor sector evolves into the infrared with non-Abelian Coulombic behavior rather than confinement and spontaneous chiral symmetry breaking. If the technicolor sector were to exhibit this behavior, it would render the model untenable, since then the only breaking of electroweak symmetry would be via QCD, at much too low a scale [1, 42].

Thus, let us consider a modification of the $N_{gen.} = 1$ model in which we change the SM-singlet fermion content so that there is a right-handed SM-singlet, HC-singlet fermion that transforms as a 3 rather than a $\bar{3}$ of $SU(3)_{ETC}$; consequently, after the breaking of $SU(3)_{ETC}$ to $SU(2)_{TC}$, it can produce a mass term for the neutrino without any ETC gauge boson mixing. To ensure that there is no $SU(3)_{ETC}$ gauge anomaly, one must also modify the other SM-singlet, ETC-nonsinglet fermion representations. An example of such a model has the SM-singlet fermion sector

$$\psi_{p,R}^i : : 3(3, 1, 1, 1)_{0,R} \quad (3.14)$$

$$\chi_{i,\alpha,R} : (\bar{3}, 2, 1, 1)_{0,R} \quad (3.15)$$

and

$$\omega_R^\alpha : (1, 2, 1, 1)_{0,R}, \quad (3.16)$$

where $p = 1, 2, 3$ is a copy index for the $\psi_{p,R}^i$ fields. The ETC symmetry breaking would again occur in the same manner as before, with obvious switches of lower and upper indices, via the formation of the condensates

$$\langle \epsilon^{1jk} \epsilon^{\alpha\beta} \chi_{j,\alpha,R}^T C \chi_{k,\beta,R} \rangle = 2 \langle \epsilon^{\alpha\beta} \chi_{2,\alpha,R}^T C \chi_{3,\beta,R} \rangle, \quad (3.17)$$

forming at the scale Λ_1 and

$$\langle \chi_{1,\alpha,R}^T C \omega_R^\alpha \rangle, \quad (3.18)$$

forming at the somewhat lower scale Λ'_1 . The fermions involved in these condensates (which include all of the HC-nonsinglet fermions) gain dynamical masses of order the respective scales Λ_1 and Λ'_1 and are integrated out in the low-energy theory that is operative below Λ'_1 .

The fermion representations of this model have been chosen so as to allow technineutrino mass terms of the form

$$- \Lambda_{TC} \sum_i \bar{n}_{i,L} \psi_{p,R}^i + h.c., \quad (3.19)$$

where the sum on i is over the TC indices, and resultant Dirac neutrino mass terms

$$- m_D \sum_i \bar{\nu}_L \psi_{p,R}^1 + h.c., \quad (3.20)$$

which could form without ETC gauge boson mixing. Of course, this unsuppressed Dirac neutrino mass term itself would be undesired without an appropriate seesaw to yield appropriately small observable neutrino masses. What we focus on here is the possible problem with this model resulting from the fact that the $SU(2)_{TC}$ sector has 18 chiral fermions, or equivalently, 9 Dirac fermions. This is slightly greater than the estimate $N_{f,cr} = 8$ for the critical number of Dirac fermions beyond which an $SU(2)$ gauge theory would evolve, in the infrared, in a non-Abelian Coulombic manner, such that the coupling would approach an infrared fixed point and never get large enough to produce spontaneous chiral symmetry breaking. Although there is a theoretical uncertainty of order $\Delta N_{f,cr} \sim 1$ in this estimate, this is a concern. In the present context, if, indeed, as the $SU(2)_{TC}$ theory evolved to scales below Λ'_1 , the coupling α_{TC} did not increase sufficiently to produce the bilinear technifermion condensates that are necessary for electroweak symmetry breaking, then the theory would not be acceptable even as a toy model of dynamical EWSB. The lesson from this model is that in constructing TC/ETC models, there is a rather tight constraint on the number of technifermions that should be present; this number should be large enough so to yield an approximate infrared fixed point in the TC beta function at a value $\alpha_{TC} = \alpha_{IR}$ that is slightly larger than the critical value α_{cr} for condensate formation, thereby giving rise to walking behavior. However, this number of technifermions must not be so large as to push α_{IR} below α_{cr} , which would cause the technicolor theory to evolve in the infrared as a non-Abelian Coulomb phase.

IV. AN ILLUSTRATIVE $N_{gen.} = 1$ MODEL WITH ONLY ETC SYMMETRIES

In this section we deviate from the use of the high-scale gauge group (2.1) and explore one of the questions posed at the beginning of the paper, namely whether, for a different value of $N_{gen.}$ than the physical value, one might be able to construct a TC/ETC model that could be simpler, in that the ETC symmetry breaking would involve only the ETC interaction itself and not have to make use of the additional strongly coupled hypercolor interaction. It is natural to investigate this question for the case of $N_{gen.} = 1$, since for this case one has what would appear to be an easier task to accomplish, namely only one, rather than three stages of ETC symmetry breaking. However, there is a countervailing effect: because the resultant ETC gauge group is just $SU(3)_{ETC}$, fermion representations that are nontrivial, distinct representations for higher $SU(N)$ groups degenerate here. This makes it more difficult to use the ETC interaction by itself to obtain condensates in channels that are more attractive than the channel in Eq. (2.23). Recall that one wants condensation in the channel (2.23) only at the low-est, technicolor, stage, not at higher ETC scales, since,

among other things, such a condensate would break electroweak symmetry at too high a scale. In the following we will use interchangeably the notation $[k]_N$ for the rank- k antisymmetric representation of $SU(N)$ and the Young tableau notation. For example, for $N_{gen.} \geq 2$ and hence, by Eq. (2.3), $N_{ETC} \geq 4$, the $[2]_{N_{ETC}}$ representation is a nontrivial distinct representation of the ETC gauge group, but for $N_{gen.} = 1$, it is equivalent to the conjugate fundamental representation, $[\bar{1}]_3$. In general, for an $SU(N)$ group with even $N = 2k$, the $[k]_N$ representation is self-conjugate. Hence, for example, for $N_{gen.} = 2$, whence $N_{ETC} = 4$, a chiral fermion ψ_R^{ij} that transforms as the $\square = [2]_4$ representation can condense as $\langle \epsilon_{ijkl} \psi_R^{ij} {}^T C \psi_R^{kl} \rangle$.

For our illustrative $N_{gen.} = 1$ model, we thus attempt to use, as the high-scale gauge group, $SU(3)_{ETC} \times G_{SM}$ without hypercolor. We choose the SM-nonsinglet fermion content as given in Eqs. (2.4) and (2.5) (with the $SU(2)_{HC}$ entries implicitly removed) and the following SM-singlet fermions:

$$\chi_{i,p,R} : 7(\bar{3}, 1, 1, 1)_0 \quad (4.1)$$

$$\psi^{ij} : (6, 1, 1, 1)_0. \quad (4.2)$$

That is, we use a fermion ψ_R^{ij} transforming as the symmetric rank-2 tensor representation of $SU(3)_{ETC}$, of dimension 6, with 7 copies of the $\bar{3}$ representation of $SU(3)_{ETC}$, indexed by $p \in \{1, \dots, 7\}$. The reason for the 7 copies is to cancel the anomaly of the symmetric tensor, which is $N + 4$ times that of the fundamental for an $SU(N)$ theory.

The first two coefficients of the beta function for this $SU(3)_{ETC}$ theory are $b_1 = 2$ and $b_2 = -79$. This two-loop perturbative beta function has an infrared zero at

$$\alpha_{ETC,IR} = \frac{8\pi}{79} \simeq 0.32. \quad (4.3)$$

Thus, as the energy scale decreases from large values, α_{ETC} increases toward this value. The most attractive channel for the formation of a bilinear fermion condensate is

$$(\bar{3}, 1, 1, 1)_0 \times (6, 1, 1, 1)_0 \rightarrow (3, 1, 1, 1)_0, \quad (4.4)$$

with $\Delta C_2 = 10/3$. If there were condensation in this channel, it would involve the condensate

$$\langle \psi_R^{ij} {}^T C \chi_{j,p,R} \rangle, \quad (4.5)$$

where, without loss of generality, we could pick $i = 1$ and $p = 1$. However, from Eq. (2.22), the minimum value of α_{ETC} for condensate formation in this channel is

$$\alpha_{cr} = \frac{\pi}{5} \simeq 0.63. \quad (4.6)$$

Even taking account of the possible contributions of higher-order terms in the beta function and the theoretical uncertainties in the analysis of the Dyson-Schwinger

equation, the value of $\alpha_{ETC,IR}$ in Eq. (4.3) is thus well below the value needed to trigger the condensation in this channel.

We conclude that as the $SU(3)_{ETC}$ theory evolves from high energy scales to lower ones, it would probably not produce any condensates and instead would probably maintain explicit chiral symmetry. The infrared zero in the $SU(3)_{ETC}$ beta function would thus be an exact infrared fixed point. Of course, the failure of the theory to break chiral symmetry with condensate formation would prevent the splitting off of SM-nonsinglet, TC-singlet components of quarks and leptons from the $SU(3)_{ETC}$ multiplets in Eqs. (2.4) and (2.5) and, related to this, would prevent the breaking of the $SU(3)_{ETC}$ symmetry to an $SU(2)_{TC}$ symmetry. The absence of any technifermion condensates breaking electroweak symmetry at the usual scale of roughly 250 GeV would exclude this theory as a useful toy model for dynamical electroweak symmetry breaking. Although we do not try to present a no-go theorem precluding the construction of a TC/ETC theory with $N_{gen.}$ SM fermion generations that could accomplish all of the stages of ETC symmetry breaking without the use of the auxiliary strongly coupled hypercolor gauge symmetry, this example shows the type of difficulties that such a construction can encounter, even for the simple case of $N_{gen.} = 1$.

V. A MODEL WITH $N_{gen.} = 2$

A. Field Content

Returning to the framework of Eq. (2.1), we next consider the case of two SM fermion generations, $N_{gen.} = 2$. Substituting this in Eq. (2.3) with $N_{TC} = 2$, we have $G_{ETC} = SU(4)_{ETC}$. In order to produce the necessary hierarchical structure for the two generations of SM fermion masses, we construct the model so that it undergoes a two-stage sequential breaking of the ETC symmetry, namely

$$SU(4)_{ETC} \rightarrow SU(3)_{ETC} \quad \text{at } \Lambda_1 \quad (5.1)$$

followed by

$$SU(3)_{ETC} \rightarrow SU(2)_{TC} \quad \text{at } \Lambda_2 \quad (5.2)$$

with $\Lambda_1 > \Lambda_2$. As before, in order to obtain this symmetry-breaking pattern, we use an auxiliary $SU(2)_{HC}$ gauge interaction. The $SU(4)_{ETC}$ gauge interaction and each of the descendants, $SU(3)_{ETC}$ and $SU(2)_{TC}$, as well as the $SU(2)_{HC}$, are asymptotically free. A choice of fermion content that can achieve the necessary two-stage breaking of the $SU(4)_{ETC}$ symmetry includes the SM-nonsinglet fermions given in Eqs. (2.4) and (2.5) with $N_{ETC} = 4$, together with the following SM-singlet fermion fields:

$$\psi_{i,R} : (\bar{4}, 1, 1, 1)_{0,R} \quad (5.3)$$

$$\chi_R^{i,\alpha} : (4, 2, 1, 1)_{0,R} \quad (5.4)$$

and

$$\zeta_R^{ij,\alpha} : (6, 2, 1, 1)_{0,R}, \quad (5.5)$$

where the numbers refer to the representations of $SU(4)_{ETC} \times SU(2)_{HC} \times SU(3)_c \times SU(2)_L$ and the subscripts denote the weak hypercharge. The 6-dimensional representation of $SU(4)$ is the antisymmetric rank-2 tensor representation, $[2]_4 = \bar{\mathbf{6}}$, which is self-conjugate (and hence has zero gauge anomaly).

The first two coefficients of the $SU(4)_{ETC}$ beta function are $b_1 = 22/3$ and $b_2 = -109/12$. Nominally, Eq. (2.19) would imply that the perturbative two-loop beta function for the $SU(4)_{ETC}$ theory has an infrared zero at $\alpha_{ETC,IR} = 352\pi/109 \simeq 10$. However, this is so large that the perturbative beta function may not be reliable. Fortunately, the only result that we need concerning this beta function is reliable, namely that the ETC coupling grows as the scale μ decreases. As will be discussed further below, the HC interaction will play the dominant role in condensate formation. The $SU(2)_{HC}$ sector has 10 chiral fermions, or equivalently, 5 Dirac fermions, transforming according to the fundamental representation. Using Eqs. (9.6) and (9.7) with $N_{f,1/2} = 2N_{f,D,1/2} = 10$, we have, for the first two coefficients of the $SU(2)_{HC}$ beta function, $b_1 = 4$ and $b_2 = 9/2$. This perturbative HC beta function does not have a zero away from the origin. Since the number $N_{f,D,1/2} = 5$ is substantially less than the estimate [23] for the critical value $N_{f,cr} \simeq 8$, we can be confident that the HC interaction does, indeed, confine and break chiral symmetry, as required for the ETC symmetry breaking.

B. Condensation at Λ_1 Breaking $SU(4)_{ETC}$ to $SU(3)_{ETC}$

To satisfy the requirement that the most attractive channels, in which the fermion condensation occurs at the high scales above the EWSB scale, are not those involving SM-nonsinglet, ETC-nonsinglet fermions, we again arrange the initial conditions specifying the strengths of the gauge couplings in the ultraviolet so that the HC gauge interaction is sufficiently stronger than the ETC interaction that the most attractive channels are those involving HC-nonsinglet (SM-singlet) fermions. Given that one arranges the model in this way, the most attractive channel for condensation is

$$(4, 2, 1, 1)_0 \times (6, 2, 1, 1)_0 \rightarrow (\bar{4}, 1, 1, 1)_0, \quad (5.6)$$

with $\Delta C_2 = C_2([2]_4) = 5/2$ for $SU(4)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon_{ijkl} \epsilon_{\alpha\beta} \chi_R^{j,\alpha} {}^T C \zeta_R^{kl,\beta} \rangle, \quad (5.7)$$

where ϵ_{ijkl} is the totally antisymmetric tensor density for $SU(4)_{ETC}$. This breaks $SU(4)_{ETC}$ to $SU(3)_{ETC}$ and

is invariant under $SU(2)_{HC}$. With no loss of generality, we may define the uncontracted $SU(4)_{ETC}$ index in Eq. (5.7) to be $i = 1$, so that this condensate is proportional to

$$\langle \epsilon_{\alpha\beta} (\chi_R^{2,\alpha} {}^T C \zeta_R^{34,\beta} - \chi_R^{3,\alpha} {}^T C \zeta_R^{24,\beta} + \chi_R^{4,\alpha} {}^T C \zeta_R^{23,\beta}) \rangle. \quad (5.8)$$

The six $\chi_R^{j,\alpha}$ with $j = 2, 3, 4$, $\alpha = 1, 2$ and the six $\zeta_R^{k\ell,\beta}$ with $k\ell = 34, 24, 23$ and $\beta = 1, 2$ involved in this condensate gain dynamical masses of order Λ_1 , and the seven ETC gauge bosons in the coset $SU(4)_{ETC}/SU(3)_{ETC}$ gain dynamical masses of order $g_{ETC}\Lambda_1$. Note that the measure of attractiveness for the channel in Eq. (5.6) with respect to the $SU(4)_{ETC}$ interaction, $\Delta C_2 = 5/2$, is less than the $\Delta C_2 = 15/4$ for the undesired condensation $4 \times \bar{4}$ involving the left- and right-handed chiral components of the HC-singlet ETC multiplets containing the SM quarks and leptons (together with the respective techniquarks and technileptons). The model is constructed so that the HC gauge coupling at the scale Λ_1 is sufficiently large that it overwhelms this difference in ΔC_2 values and makes Eq. (5.6) the most attractive channel.

C. Theory for $\Lambda_2 \leq E < \Lambda_1$ and Condensation at Λ_2 Breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$

In the low-energy effective field theory operative at energy scales μ directly below Λ_1 , the light fermions that are nonsinglets under the strongly coupled $SU(3)_{ETC}$ and/or $SU(2)_{HC}$ gauge groups include the SM nonsinglets in Eqs. (2.4) and (2.5) and the SM singlets (i) $\zeta_R^{1j,\alpha}$, (ii) $\chi_R^{1,\alpha}$, and (iii) $\psi_{j,R}$ with $j = 2, 3, 4$. These transform, respectively as (i) $(\bar{3}, 2)$, (ii) $(1, 2)$, and (iii) $(\bar{3}, 1)$ representations of $SU(3)_{ETC} \times SU(2)_{HC}$. As the theory evolves to lower energy scales μ , the ETC and HC gauge couplings continue to grow, and as μ decreases through a scale that we denote Λ_2 , the dominant $SU(2)_{HC}$ interaction, in conjunction with the additional strong $SU(3)_{ETC}$ interaction, produces a condensate in the most attractive channel, which is

$$(3, 2, 1, 1)_0 \times (3, 2, 1, 1)_0 \rightarrow (\bar{3}, 1, 1, 1)_0. \quad (5.9)$$

This has $\Delta C_2 = 4/3$ for $SU(3)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. The condensation in this channel breaks $SU(3)_{ETC}$ to $SU(2)_{TC}$ and is invariant under $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon_{ijk} \epsilon_{\alpha\beta} \zeta_R^{1j,\alpha} {}^T C \zeta_R^{1k,\beta} \rangle, \quad (5.10)$$

where $i, j, k \in \{2, 3, 4\}$. With no loss of generality, we may choose $i = 2$ as the breaking direction in $SU(3)_{ETC}$, so that this condensate takes the form

$$2 \langle \epsilon_{\alpha\beta} \zeta_R^{13,\alpha} {}^T C \zeta_R^{14,\beta} \rangle. \quad (5.11)$$

The four chiral fermions $\zeta_R^{13,\alpha}$ and $\zeta_R^{14,\alpha}$ with $\alpha = 1, 2$ gain masses of order Λ_2 , and the five ETC gauge bosons in the coset $SU(3)_{ETC}/SU(2)_{TC}$ gain masses of order $g_{ETC}\Lambda_2$.

D. Condensation at $\Lambda'_2 < \Lambda_2$

The low-energy effective field theory operative just below Λ_2 is thus invariant under the direct product group

$$SU(2)_{TC} \times SU(2)_{HC} \times G_{SM}. \quad (5.12)$$

The massless SM-singlet fermions that are nonsinglets under $SU(2)_{TC}$ or $SU(2)_{HC}$ are $\chi_R^{1,\alpha}$, $\zeta_R^{12,\alpha}$, and $\psi_{i,R}$ with $i = 3, 4$. The first two of these transform as $(1, 2)$ under $SU(2)_{TC} \times SU(2)_{HC}$ and the last as $(\bar{2}, 1) \approx (2, 1)$. Given that $\alpha_{HC} > \alpha_{TC}$, the most attractive channel is $(1, 2) \times (1, 2) \rightarrow (1, 1)$, with condensate

$$\langle \epsilon_{\alpha\beta} \chi_R^{1,\alpha} {}^T C \zeta_R^{12,\beta} \rangle, \quad (5.13)$$

with $\Delta C_2 = 3/2$. The condensate is invariant under the full group (5.12). It forms at a scale $\Lambda'_2 < \Lambda_2$, since it has the same measure of attractiveness, ΔC_2 , with respect to $SU(2)_{HC}$ as the channel (5.9), but is driven by hypercolor alone, while the channel (5.9) involves attraction due to both $SU(2)_{HC}$ and $SU(3)_{ETC}$. The four chiral fermions $\chi_R^{1,\alpha}$ and $\zeta_R^{12,\alpha}$ get dynamical masses of order Λ'_2 .

E. Theory at Energy Scales Below Λ'_2

In the low-energy theory below Λ'_2 , all of the HC-nonsinglet fermions have gained dynamical masses and have consequently been integrated out. The fermions in Eqs. (2.4) and (2.5) together with $\psi_{i,R}$ with $i = 3, 4$ comprise 16 chiral doublets, or equivalently, 8 Dirac doublets, of $SU(2)_{TC}$. As noted before, this TC theory plausibly exhibits walking behavior. The Dirac mass terms for the technineutrinos are of the form (2.13). The $\psi_{i,R}$ with $i = 1, 2$ are TC-singlets that play the role of electroweak-singlet neutrinos.

Apart from mixing effects, the SM fermion masses of generation i are generically of the form

$$m_{f^i} \simeq \kappa \eta_i \frac{\Lambda_{TC}^3}{\Lambda_i^2}, \quad i = 1, 2, \quad (5.14)$$

where the renormalization factor η_i is given by

$$\eta_i = \exp \left[\int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha(\mu)) \right]. \quad (5.15)$$

Here, as before, α refers to the TC coupling as inherited from the enveloping ETC theory. (The generational index i is written as an upper, rather than lower, index on f^i because the quarks and charged leptons arise from

fundamental, rather than conjugate fundamental, representations of the ETC group.) Assuming that the theory exhibits walking up to Λ'_2 , this would be roughly of order

$$\eta_i \sim \frac{\Lambda'_2}{\Lambda_{TC}} \quad (5.16)$$

for both $i = 1$ and $i = 2$. It follows that, again neglecting mixing,

$$m_{f^1} \simeq \kappa \frac{\Lambda'_2 \Lambda_{TC}^2}{\Lambda_1^2} \quad (5.17)$$

and

$$m_{f^2} \simeq \kappa \frac{\Lambda'_2 \Lambda_{TC}^2}{\Lambda_2^2}, \quad (5.18)$$

so that

$$\frac{m_{f^1}}{m_{f^2}} = \left(\frac{\Lambda_2}{\Lambda_1} \right)^2. \quad (5.19)$$

Thus, this model succeeds in producing a generational hierarchy in the standard-model fermion masses. Furthermore, because of the expected walking behavior of the technicolor sector and the resultant enhancement of fermion masses via the factor (5.16), the fermions of the higher generation, $i = 2$, could have masses that are not too much smaller than the electroweak symmetry breaking scale, Λ_{TC} .

The various condensates of ETC-nonsinglet fermions give rise to corrections to fermion propagators that are nondiagonal in ETC indices. In turn, via vacuum polarization diagrams, these produce mixings of different ETC gauge bosons. In Figs. 2 and 3 we show graphs contributing to the ETC gauge boson mixing $V_1^3 \leftrightarrow V_4^1$, or equivalently, $V_1^4 \leftrightarrow V_3^1$. However, we find that the ETC gauge boson mixing is not sufficient to give rise to the neutrino seesaw mechanism of Ref. [10], so one would have to add further ingredients to the model in order to obtain appropriately small nonzero neutrino masses.

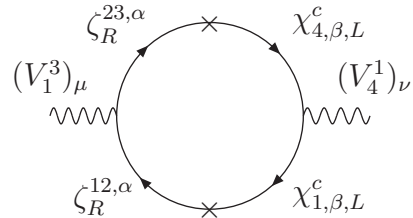


FIG. 2: One-loop graph contributing to the ETC gauge boson mixing $V_1^3 \leftrightarrow V_4^1$ (equivalently, $V_1^4 \leftrightarrow V_3^1$) in the $N_{gen.} = 2$ model.

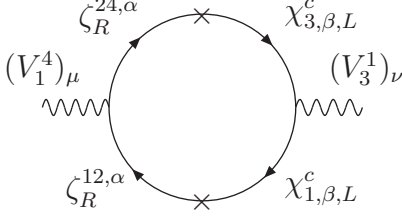


FIG. 3: One-loop graph contributing to the ETC gauge boson mixing $V_1^4 \leftrightarrow V_3^1$ (equivalently, $V_1^1 \leftrightarrow V_4^1$) in the $N_{gen.} = 2$ model.

VI. A MODEL WITH $N_{gen.} = 3$

A. Field Content

Here, for reference, we give a brief review of an $N_{gen.} = 3$ model, which is one of the models studied in Refs. [4, 10, 12]. With $N_{gen.} = 3$ and $N_{TC} = 2$, one uses $SU(5)_{ETC}$ for the ETC group. The SM-nonsinglet fermions and technifermions are given in Eqs. (2.4) and (2.5). The SM-singlet fermions are

$$\psi_{ij,R} : (\overline{10}, 1, 1, 1)_{0,R} \quad (6.1)$$

$$\zeta_R^{ij,\alpha} : (10, 2, 1, 1)_{0,R} \quad (6.2)$$

and

$$\omega_{\alpha,p,R} : 2(1, 2, 1, 1)_{0,R}, \quad (6.3)$$

where the first number in parentheses is the dimension of the representation of $SU(5)_{ETC}$ and the others are the same as defined before. In Eq. (6.3), $p = 1, 2$ is the copy number for the $\omega_{\alpha,p,R}$ fields. The $[2]_5$ is the rank-2 antisymmetric representation of $SU(5)$, with dimension 10. One includes an even number of copies of the $\omega_{\alpha,p,R}$ field in order to avoid a global anomaly in the $SU(2)_{HC}$ theory, and the choice of two copies is made to produce desired mixings of ETC gauge bosons and resultant off-diagonal elements of fermion mass matrices.

The $SU(5)_{ETC}$ beta function of this theory has leading coefficients $b_1 = 31/3$ and $b_2 = 224/15$. Thus, the coupling α_{ETC} increases to large values as the energy scale decreases, triggering the formation of condensates. The $SU(2)_{HC}$ sector has 12 chiral fermions, or equivalently, 6 Dirac fermions, transforming according to the fundamental representation. From Eqs. (9.6) and (9.7) with $N_{f,1/2} = 2N_{f,D,1/2} = 12$, it follows that the first two coefficients of the $SU(2)_{HC}$ beta function are $b_1 = 10/3$ and $b_2 = -11/3$. The number $N_{f,D,1/2} = 6$ is less than the estimated critical value of Dirac fermions, $N_{f,cr} \simeq 8$, leading to the inference that the HC interaction confines and produces chiral condensates.

B. Condensation at Λ_1 Breaking $SU(5)_{ETC}$ to $SU(4)_{ETC}$

In the context of variable $N_{gen.}$, one can comment on some features that are present in this $N_{gen.} = 3$ case that were not present for the lower values of $N_{gen.}$. Notably, for $SU(5)$, fermions in the conjugate rank-2 antisymmetric $\overline{10}$ representation can play an important role in self-breaking of the ETC symmetry. In contrast, for $SU(3)$ the rank-2 antisymmetric representation is equivalent to a conjugate fundamental representation, while for $SU(4)$ it is self-conjugate. For $SU(5)$, the $\overline{10}$ can form a condensate with itself via the channel

$$(\overline{10}, 1, 1, 1)_0 \times (\overline{10}, 1, 1, 1)_0 \rightarrow (5, 1, 1, 1)_0. \quad (6.4)$$

The attractiveness of this channel, given by $\Delta C_2 = 24/5$, is the same as for the undesired channel condensation (2.23). One can invoke vacuum alignment arguments to infer that the initial condensation will occur in the channel (6.4) rather than (2.23). As was noted in Ref. [10], the channel (6.4) is actually not the channel with the largest value of ΔC_2 ; the latter is $(\overline{10}, 1, 1, 1)_0 \times (10, 2, 1, 1)_0 \rightarrow (1, 2, 1, 1)_0$, with $\Delta C_2 = 36/5$, which would leave $SU(5)_{ETC}$ invariant and would break $SU(2)_{HC}$. One must thus invoke a vacuum alignment and generalized most attractive channel argument to infer that the latter condensation does not occur, since it would break the strongly coupled HC interaction [10]. The formation of a condensate in the channel (6.4) breaks $SU(5)_{ETC}$ to $SU(4)_{ETC}$. Hence, in this $N_{gen.} = 3$ case, in contrast to the situation for the $N_{gen.} = 1, 2$ models analyzed above in Sections III and V, one can use the ETC interaction for the first stage of ETC gauge symmetry breaking. Thus, here, the $SU(5)_{ETC}$ symmetry self-breaks, while in the $N_{gen.} = 1, 2$ models, the breaking of ETC is caused primarily by the HC interaction. The scale at which the condensate (6.4) forms is denoted as Λ_1 . Choosing, as before, the direction of breaking to be $i = 1$, one obtains the condensate $\langle \epsilon^{1jkl} \psi_{jk,R}^T C \psi_{ln,R} \rangle$, or equivalently,

$$\langle \psi_{23,R}^T C \psi_{45,R} - \psi_{24,R}^T C \psi_{35,R} + \psi_{25,R}^T C \psi_{34,R} \rangle. \quad (6.5)$$

The six components $\psi_{jk,R}$ involved in this condensate gain dynamical masses of order Λ_1 . The nine ETC gauge bosons in the coset $SU(5)_{ETC}/SU(4)_{ETC}$ gain masses of order $g_{ETC} \Lambda_1 \sim \Lambda_1$. The components of the multiplets in Eqs. (2.4) and (2.5) with $i = 1$ split off from the other components and become the first generation of SM fermions.

C. Theory for $\Lambda_2 \leq E < \Lambda_1$ and Condensation at Λ_2 Breaking $SU(4)_{ETC}$ to $SU(3)_{ETC}$

The low-energy effective field theory just below Λ_1 is invariant under two strongly coupled gauge symmetries, $SU(4)_{ETC}$, acting on the ETC indices $2 \leq i \leq 5$,

and $SU(2)_{HC}$. Decomposing the massless fermions inherited from the $SU(5)_{ETC}$ theory in terms of representations of $SU(4)_{ETC}$ (and the other exact symmetries at this level), one has the following content: (i) $\psi_{1j,R}$, a $(\bar{4}, 1, 1, 1)_0$; (ii) $\zeta_R^{1j,\alpha}$, a $(4, 2, 1, 1)_0$; (iii) $\zeta_R^{jk,\alpha}$, a $(6, 2, 1, 1)_0$; and (iv) $\omega_{\alpha,p,R}$, forming two $(1, 2, 1, 1)_0$ representations, where the $SU(4)_{ETC}$ gauge indices are $2 \leq i, j \leq 5$, the $SU(2)_{HC}$ indices are $\alpha = 1, 2$, and the copy index is $p = 1, 2$. The next two stages of ETC symmetry breaking involve both the ETC and the HC interactions. With the HC interaction sufficiently strong, the next preferred step in gauge symmetry breaking, occurring at the scale Λ_2 , involves the formation of a condensate in the most attractive channel

$$(4, 2, 1, 1)_0 \times (6, 2, 1, 1)_0 \rightarrow (\bar{4}, 1, 1, 1)_0, \quad (6.6)$$

with $\Delta C_2 = 5/2$ for $SU(4)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. This breaks $SU(4)_{ETC}$ to $SU(3)_{ETC}$ and pre-

serves the exact $SU(2)_{HC}$ symmetry. Given that the $SU(2)_{HC}$ interaction is strongly coupled at Λ_2 , the HC glueballs are expected to have masses of order $\kappa_g \Lambda_2$.

The symmetry breaking pattern in which this is the second stage was denoted G_a in Ref. [10] and sequence S_1 in Ref. [12]. Note that, with respect to the $SU(4)_{ETC}$ interaction, the value of ΔC_2 for this channel is less than the value $\Delta C_2 = 15/4$ for the undesired $4 \times \bar{4} \rightarrow 1$ channel (2.23) involving SM-nonsinglet fermions. Thus, one again specifies a sufficiently large initial value for the HC coupling α_{HC} at a high scale so that the combination of the HC and ETC interactions renders the channel (6.6) more attractive than the channel $4 \times \bar{4} \rightarrow 1$ channel. With no loss of generality, one defines the index in which the $SU(4)_{ETC}$ breaks as $i = 2$, so that the condensate is

$$\langle \epsilon_{\alpha\beta} \epsilon_{2jkl} \zeta_R^{1j,\alpha} {}^T C \zeta_R^{kl,\beta} \rangle = 2 \langle \epsilon_{\alpha\beta} (\zeta_R^{13,\alpha} {}^T C \zeta_R^{45,\beta} - \zeta_R^{14,\alpha} {}^T C \zeta_R^{35,\beta} + \zeta_R^{15,\alpha} {}^T C \zeta_R^{34,\beta}) \rangle. \quad (6.7)$$

Here, ϵ_{ijkl} is the totally antisymmetric tensor density of the $SU(4)_{ETC}$ theory resulting from the breaking of $SU(5)_{ETC}$ and hence is identical to ϵ_{ijkl} of the $SU(5)_{ETC}$ theory. The twelve $\zeta_R^{ij,\alpha}$ fields in this condensate gain masses of order Λ_2 , and the seven ETC gauge bosons in the coset $SU(4)_{ETC}/SU(3)_{ETC}$ gain masses of order $g_{ETC} \Lambda_2 \simeq \Lambda_2$. At this scale Λ_2 , the second generation SM fermions, with $i = 2$, split off from the other components of the multiplets in Eqs. (2.4) and (2.5).

D. Theory for $\Lambda_3 \leq E < \Lambda_2$ and Condensation at Λ_3 Breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$

Because the effective field theory below Λ_2 has $SU(3)_{ETC}$ symmetry (acting on the ETC indices $i = 3, 4, 5$), to analyze this, we decompose the $SU(4)_{ETC}$ representations in terms of $SU(3)_{ETC}$. The massless fermions that are nonsinglets under the ETC and/or HC groups operative here are (i) $\psi_{1j,R}$, a $(\bar{3}, 1, 1, 1)_0$; (ii) $\zeta_R^{2j,\alpha}$, a $(3, 2, 1, 1)_0$ where $j = 3, 4, 5$; (iii) $\zeta_R^{12,\alpha}$, a $(1, 2, 1, 1)_0$; and (iv) $\omega_{\alpha,p,R}$, comprising two $(1, 2, 1, 1)_0$ representations. Since the $SU(3)_{ETC}$ and $SU(2)_{HC}$ interactions are asymptotically free, α_{ETC} and α_{HC} continue to grow. A third and final stage of ETC symmetry breaking occurs at a scale denoted Λ_3 . Given the specification of the strengths of the ETC and HC interaction, the most attractive channel is

$$(3, 2, 1, 1)_{0,R} \times (3, 2, 1, 1)_{0,R} \rightarrow (\bar{3}, 1, 1, 1)_0, \quad (6.8)$$

with $\Delta C_2 = 4/3$ for $SU(3)_{ETC}$ and $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. This breaks $SU(3)_{ETC}$ to $SU(2)_{TC}$ and preserves $SU(2)_{HC}$. With the breaking direction taken as $i = 3$, the associated condensate is

$$\langle \epsilon_{3jk} \epsilon_{\alpha\beta} \zeta_R^{2j,\alpha} {}^T C \zeta_R^{2k,\beta} \rangle = 2 \langle \zeta_R^{24,1} {}^T C \zeta_R^{25,2} - \zeta_R^{24,2} {}^T C \zeta_R^{25,1} \rangle. \quad (6.9)$$

Here, ϵ_{ijk} is the totally antisymmetric tensor density of the $SU(3)_{ETC}$ theory resulting from the breaking of $SU(4)_{ETC}$ and hence is identical to ϵ_{2ijk} of the $SU(4)_{ETC}$ theory. The six $\zeta_R^{2j,\alpha}$ fields involved in this condensate gain dynamical masses of order Λ_3 , and the five ETC gauge bosons in the coset $SU(3)_{ETC}/SU(2)_{TC}$ gain masses of order $g_{ETC} \Lambda_3 \simeq \Lambda_3$. At this scale Λ_3 , the third generation of SM fermions splits off, leaving the residual technifermions in each of the respective multiplets.

E. Condensation at $\Lambda'_3 < \Lambda_3$

Since the HC interaction is strong, it can also produce condensates involving residual massless fermions that are singlets under $SU(2)_{TC}$ but nonsinglets under $SU(2)_{HC}$, in the channel

$$(1, 2, 1, 1)_0 \times (1, 2, 1, 1)_0 \rightarrow (1, 1, 1, 1)_0, \quad (6.10)$$

where the first number is the dimension of the representation with respect to $SU(3)_{ETC}$ and the others are as before. The condensates that form in this channel include

$$\langle \epsilon_{\alpha\beta} \zeta_R^{12,\alpha} {}^T C \zeta_R^{23,\beta} \rangle, \quad (6.11)$$

$$\langle \epsilon_{\alpha\beta} \zeta_R^{12,\alpha} {}^T C \omega_{p,R}^\beta \rangle, \quad (6.12)$$

$$\langle \epsilon_{\alpha\beta} \zeta_R^{23,\alpha} {}^T C \omega_{p,R}^\beta \rangle, \quad (6.13)$$

and

$$\langle \epsilon_{\alpha\beta} \omega_{1,R}^\alpha {}^T C \omega_{2,R}^\beta \rangle \quad (6.14)$$

with $p = 1, 2$. Since these condensates are formed only via the hypercolor attraction, without any additional $SU(3)_{ETC}$ interaction, they form at a scale $\Lambda'_3 \lesssim \Lambda_3$, where α_{HC} has grown somewhat larger than at Λ_3 . The fermions involved in these condensates get dynamical masses of order Λ'_3 .

F. Theory at Energy Scales Below Λ'_3

In the effective theory below Λ_3 , all of the fermions $\zeta_R^{ij,\alpha}$ have gained masses and have been integrated out, as have all of the $\psi_{ij,R}$ for $2 \leq i, j \leq 5$, and all of the $\omega_{\alpha,p,R}$ fields. The resultant theory has one strongly coupled symmetry with massless nonsinglet fermions, namely the technicolor group $SU(2)_{TC}$. The technifermions include those with SM-nonsinglet quantum numbers, given in Eq. (2.4) and (2.5), and the $\psi_{1j,R}$ for $j = 4, 5$. These constitute $4(N_c + 1) = 16$ chiral fermion doublets, or equivalently, 8 Dirac fermion doublets, of $SU(2)_{TC}$. Thus, $b_1 = 2$ and $b_2 = -20$ for the beta function of this theory, which has an approximate infrared zero at $\alpha_{TC} = 2\pi/5 \simeq 1.3$. To within the theoretical uncertainties, this is equal to the critical value $\alpha_{cr} = 4\pi/9 \simeq 1.4$ for condensation of the technifermions to form the condensates (2.12). Furthermore, as noted, since the number of technifermions in this theory is close to $N_{f,cr}$, it plausibly exhibits the desired property of walking associated with the approximate infrared fixed point at α_{TC} .

The various condensates give rise to a variety of ETC gauge boson mixings. In turn, these lead to mass matrices for the quarks and charged leptons with both hierarchical diagonal and off-diagonal entries. The diagonal elements have the generic form of Eq. (5.14) with $i = 1, 2, 3$. Since the walking behavior extends over the energy interval where there are eight massless Dirac technifermions, namely from Λ_{TC} to Λ_3 , it follows that the renormalization factor is roughly $\eta_i \sim \Lambda_3/\Lambda_{TC}$. This makes it possible for the mass of the top quark to be comparable to the electroweak symmetry-breaking scale. However, as noted, the model with the content of quarks, leptons, techniquarks, and technileptons as given in Eqs. (2.4) and (2.5) has difficulty explaining the large splitting between m_t and m_b . In general, the running masses of SM fermions of the i 'th generation, $m_{fi}(p)$, are constants (apart from logs) up to the highest scale, Λ_i , where they arise, and decay asymptotically like $m_{fi}(p) \propto \Lambda_i^2/p^2$ (up to logs) for $p \gg \Lambda_i$, where p is a Euclidean momentum [29]. This model is also able to produce appropriate Dirac and Majorana masses for neutrinos, in a manner

such as to yield a seesaw that generates acceptably light observed neutrino mass eigenstates [10, 12]. Further details are given in Refs. [4, 10, 12, 14].

VII. A MODEL WITH $N_{gen.} = 4$

A. Field Content

One can also investigate a situation with $N_{gen.}$ larger than the inferred physical value of 3 [8]. In considering models with larger values of $N_{gen.}$, we will require that these models retain the basic properties of QCD, namely that (i) it is asymptotically free, which implies that the number of quarks, N_q , is bounded above by $N_q < 33/2$; and (ii) as the scale decreases below a GeV, QCD should confine and spontaneously break chiral symmetry rather than evolving into the infrared in a chirally symmetric manner such as would be associated with a non-Abelian Coulomb phase. Analyses of the Dyson-Schwinger equation for the quark propagator [23] yield $N_{f,cr} \simeq 12$, and recent lattice simulations are broadly consistent with this estimate, to within their theoretical uncertainties [25]. Keeping the number of quarks below 12 means keeping the number of generations below 6, which allows one to consider the values $N_{gen.} = 4$ and $N_{gen.} = 5$, given the above constraints. Here we will study the case $N_{gen.} = 4$. As noted before, although we will consider this case from the abstract field-theoretic point of view of its effect in constructing a TC/ETC model, we note that there are continuing studies of the possibility that there really are four generations of SM fermions, which, however, must necessarily avoid having a fourth light active neutrino [15]–[18]. Combining the value $N_{gen.} = 4$ with the value $N_{TC} = 2$ in Eq. (2.3) yields $SU(6)_{ETC}$ as the ETC gauge group. Just as for $N_{gen.} = 3$ one could use purely ETC interactions for the first stage of ETC symmetry breaking (which is thus self-breaking), so also for this $N_{gen.} = 4$ case we find that one can use the ETC interaction by itself for the first two stages of ETC self-breaking, from $SU(6)_{ETC}$ to $SU(5)_{ETC}$ and then to $SU(4)_{ETC}$. We rely on the hypercolor interaction to produce the final two stages of ETC breaking down to $SU(2)_{TC}$. However, we specify the initial value of the HC coupling slightly above the first condensation to be such that as the HC coupling α_{HC} grows, it becomes significantly large at the third level of symmetry breaking, Λ_3 .

We take the SM-singlet chiral fermions of the model to consist of

$$\chi_R^i : ([1]_6, 1, 1, 1)_0 \quad (7.1)$$

$$\psi_R^{ij} : ([2]_6, 1, 1, 1)_0 \quad (7.2)$$

$$\eta_R^{ijk} : ([3]_6, 1, 1, 1)_0 \quad (7.3)$$

$$\zeta_{i,\alpha,R} : ([\bar{1}]_6, 2, 1, 1)_0 \quad (7.4)$$

and

$$\omega_{\alpha,p,R} : 2(1, 2, 1, 1)_0, \quad (7.5)$$

where the $SU(6)_{ETC}$ gauge indices run from 1 to 6, and the copy index on $\omega_{\alpha,p,R}$ takes the values $p = 1, 2$. Note that since the $\zeta_{i,\alpha,R}$ fields comprise an even number (six) of $SU(2)_{HC}$ doublets, it is necessary to use an even number of the ETC-singlet, HC-doublet $\omega_{\alpha,p,R}$ fields to avoid a global $SU(2)$ anomaly. We use two copies, $p = 1, 2$. Since the dimensionality of the $[k]_N$ representation is $\binom{N}{k}$, we have $\dim([1]_6) = 6$, $\dim([2]_6) = 15$, and $\dim([3]_6) = 20$. By construction, this theory is free of anomalies in the $SU(6)$ gauged currents. The one- and two-loop coefficients of the $SU(6)_{ETC}$ beta function are

$$b_1 = \frac{38}{3}, \quad b_2 = \frac{76}{3}. \quad (7.6)$$

Since these have the same sign, the $SU(6)_{ETC}$ beta function does not have a perturbative zero away from the origin. The $SU(2)_{HC}$ theory contains eight chiral fermions, or equivalently four Dirac fermions, transforming as doublet representations. This is well below the estimated value $N_{f,cr} \simeq 8$ separating the chirally broken from chirally symmetric phases of an $SU(2)$ theory, so that we can be confident that the $SU(2)_{HC}$ interaction confines and produces fermion condensates as required.

B. Condensation at Λ_1 Breaking $SU(6)_{ETC}$ to $SU(5)_{ETC}$

The most attractive channel and hence the one most likely for condensation to form at the highest scale, is

$$([2]_6, 1, 1, 1)_0 \times ([3]_6, 1, 1, 1)_0 \rightarrow ([\bar{1}]_6, 1, 1, 1)_0 \quad (7.7)$$

with $\Delta C_2 = 7$, which breaks $SU(6)_{ETC}$ to $SU(5)_{ETC}$. As before, we denote this first and highest ETC symmetry-breaking scale as Λ_1 . Note that this is more attractive than the undesired condensation channel $6 \times \bar{6} \rightarrow 1$ involving the left- and right-handed components of the SM-nonsinglet fermions in Eqs. (2.4) and (2.5), which has $\Delta C_2 = 35/6 = 5.83$. The condensate associated with the channel (7.7) is

$$\langle \epsilon_{ijklmn} \psi_R^{jk} C^T \eta_R^{\ell mn} \rangle, \quad (7.8)$$

where ϵ_{ijklmn} is the totally antisymmetric tensor density for $SU(6)$. With no loss of generality, we can pick the uncontracted $SU(6)_{ETC}$ index to be $i = 1$. The $\binom{5}{2} = 10$ components ψ_R^{jk} with $2 \leq j, k \leq 6$ and $j \neq k$, and the $\binom{5}{3} = 10$ components $\eta_R^{\ell mn}$ with $2 \leq \ell, m, n \leq 6$ (with unequal values of ℓ, m, n) pick up dynamical masses of order Λ_1 . The 11 ETC gauge bosons in the coset $SU(6)_{ETC}/SU(5)_{ETC}$ also gain masses $\sim g_{ETC} \Lambda_1 \sim \Lambda_1$. At this stage, χ_R^1 decouples from the strong dynamics, since it is a singlet under the residual $SU(5)_{ETC} \times SU(2)_{HC}$ interaction.

C. Theory for $\Lambda_2 \leq E < \Lambda_1$ and Condensation at Λ_2 Breaking $SU(5)_{ETC}$ to $SU(4)_{ETC}$

The low-energy theory operative just below Λ_1 has two strongly coupled gauge groups, $SU(5)_{ETC}$ and $SU(2)_{HC}$. The content of massless SM-singlet fermions that are nonsinglets under these groups, in addition to $\omega_{\alpha,R}$, is

$$\chi_R^j : (5, 1, 1, 1)_0 \quad (7.9)$$

$$\psi_R^{1j} : (5, 1, 1, 1)_0 \quad (7.10)$$

and

$$\zeta_{j,\alpha,R} : (\bar{5}, 2, 1, 1)_0 \quad (7.11)$$

with $2 \leq j \leq 6$, and

$$\eta_R^{1jk} : (10, 1, 1, 1)_0, \quad (7.12)$$

where $2 \leq j \neq k \leq 6$. A most attractive channel in this theory is

$$(10, 1, 1, 1)_0 \times (10, 1, 1, 1)_0 \rightarrow (\bar{5}, 1, 1, 1)_0 \quad (7.13)$$

with $\Delta C_2 = 24/5$, breaking $SU(5)_{ETC}$ to $SU(4)_{ETC}$. With no loss of generality, we may take the breaking direction in $SU(5)_{ETC}$ to be $i = 2$, so that associated condensate is

$$\langle \epsilon_{2jklm} \eta_R^{1jk} C^T \eta_R^{1\ell m} \rangle, \quad (7.14)$$

where ϵ_{ijklm} is the totally antisymmetric tensor density of $SU(5)_{ETC}$ and the sums over repeated indices are over the values 3, 4, 5, and 6. Since the measure of attractiveness for this condensation channel, $\Delta C_2 = 24/5$, is less than the $\Delta C_2 = 7$ for the first condensation, it follows that the scale, Λ_2 , at which the second condensation occurs is lower than the first condensation scale, Λ_1 . As in the $N_{gen.} = 3$ case, one can use vacuum alignment arguments to infer that the condensation (7.13) occurs instead of the unwanted condensation (2.23) involving the SM-nonsinglet fermions (which has the same value of $\Delta C_2 = 24/5$). The six chiral fermions η_R^{1jk} with $3 \leq j, k \leq 6$ involved in the condensate (7.14) gain dynamical masses of order Λ_2 , and the nine ETC gauge bosons in the coset $SU(5)/SU(4)_{ETC}$ gain masses of order $g_{ETC} \Lambda_2 \sim \Lambda_2$. At this stage, χ_R^2 and ψ_R^{12} decouple from the strong dynamics, since they are singlets under the residual $SU(4)_{ETC} \times SU(2)_{HC}$ interaction. Note that, as far as $SU(5)_{ETC}$ is concerned, the channel $(10, 2, 1, 1)_0 \times (10, 2, 1, 1)_0 \rightarrow (\bar{5}, 3, 1, 1)_0$ is as attractive as the channel (7.13), with $\Delta C_2 = 24/5$. However, since this involves a symmetric combination of the two representations, it yields a triplet representation under $SU(2)_{HC}$ and is thus repulsive with respect to the HC interaction, with $\Delta C_2 = -1/2$. For this reason, one may safely conclude that there is no condensation in this channel.

D. Theory for $\Lambda_3 \leq E < \Lambda_2$ and Condensation at Λ_3 Breaking $SU(4)_{ETC}$ to $SU(3)_{ETC}$

The low-energy effective theory operative just below the scale Λ_2 is invariant under two strongly coupled groups, $SU(4)_{ETC} \times SU(2)_{HC}$, where the $SU(4)_{ETC}$ acts on the indices $3 \leq j \leq 6$. The content of massless fermions that are nonsinglets under this direct product group includes

$$\chi_R^j : (4, 1, 1, 1)_0 \quad (7.15)$$

$$\psi_R^{1j} : (4, 1, 1, 1)_0 \quad (7.16)$$

$$\zeta_{j,\alpha,R} : (\bar{4}, 2, 1, 1)_0 \quad (7.17)$$

and

$$\eta_R^{12j} : (4, 1, 1, 1)_0, \quad (7.18)$$

with $3 \leq j \leq 6$. There are also massless fermions that are singlets under $SU(4)_{ETC}$ and doublets under $SU(2)_{HC}$ namely $\zeta_{j,\alpha,R}$, $j = 1, 2$ and $\omega_{\alpha,p,R}$. Since the hypercolor interaction is asymptotically free, its coupling, α_{HC} , increases as the reference scale μ decreases from Λ_1 through Λ_2 , and the initial conditions can be chosen so that at the scale $\Lambda_3 < \Lambda_2$, the $SU(2)_{HC}$ interaction is sufficiently strong to produce condensation of HC-doublet fermions. The most attractive channel is

$$(\bar{4}, 2, 1, 1)_0 \times (1, 2, 1, 1)_0 \rightarrow (\bar{4}, 1, 1, 1)_0, \quad (7.19)$$

with $\Delta C_2 = 3/2$ for $SU(3)_{HC}$. This breaks $SU(4)_{ETC}$ to $SU(3)_{ETC}$ and preserves $SU(2)_{HC}$. The associated condensate is

$$\langle \epsilon^{\alpha\beta} \zeta_{j,\alpha,R}^T C \omega_{p,\beta,R} \rangle. \quad (7.20)$$

With no loss of generality, we choose the ETC gauge index $j = 3$ so that the residual $SU(3)_{ETC}$ gauge symmetry acts on the indices 4, 5, and 6. We may also choose the copy index to be $p = 1$ for the $\omega_{\alpha,p,R}$ field. The explicit condensate is then

$$\langle \epsilon^{\alpha\beta} \zeta_{3,\alpha,R}^T C \omega_{1,\beta,R} \rangle. \quad (7.21)$$

The $\zeta_{3,\alpha,R}$ and $\omega_{1,\beta,R}$ fields pick up dynamical masses of order Λ_3 and the ETC five gauge bosons in the coset $SU(3)_{ETC}/SU(2)_{TC}$ gain masses of order $g_{ETC} \Lambda_3 \sim \Lambda_3$.

With the same degree of attractiveness, and hence at the same scale, Λ_3 , there is an HC-driven condensation of two $SU(4)_{ETC}$ -singlet fields in the channel $(1, 2, 1, 1)_0 \times (1, 2, 1, 1)_0 \rightarrow (1, 1, 1, 1)_0$. The associated condensate is

$$\langle \epsilon^{\alpha\beta} \zeta_{1,\alpha,R}^T C \zeta_{2,\beta,R} \rangle. \quad (7.22)$$

This is invariant under the same strongly coupled $SU(3)_{ETC} \times SU(2)_{HC}$ symmetry group as the condensate (7.20). As a consequence of these condensations, the fermions $\zeta_{j,\alpha,R}$ with $j = 1, 2, 3$ and $\omega_{\alpha,R}$ gain dynamical masses of order Λ_3 and the seven ETC gauge bosons in the coset $SU(4)_{ETC}/SU(3)_{ETC}$ gain masses of order $g_{ETC} \Lambda_3 \sim \Lambda_3$. At this stage, χ_R^3 , ψ_R^{13} , and η_R^{123} decouple from the strong dynamics since they are singlets under the residual $SU(3)_{ETC} \times SU(2)_{ETC}$ interaction.

E. Theory for $\Lambda_4 \leq E < \Lambda_3$ and Condensation at Λ_4 Breaking $SU(3)_{ETC}$ to $SU(2)_{TC}$

The effective field theory operative just below Λ_3 is invariant under the strongly coupled group $SU(3)_{ETC} \times SU(2)_{HC}$, with the $SU(3)_{ETC}$ acting on the indices $j = 4, 5$, and 6. The massless fermions that are nonsinglets under this group are (i) χ_R^j , ψ_R^{1j} , and η_R^{12j} , forming $(3, 1, 1, 1)_0$ representations; (ii) $\zeta_{j,\alpha,R}$, forming $(\bar{3}, 2, 1, 1)_0$; and (iii) $\omega_{\alpha,2,R}$, forming $(1, 2, 1, 1)_0$. The most attractive channel, which involves both $SU(3)_{ETC}$ and $SU(2)_{HC}$ interactions, is

$$(\bar{3}, 2, 1, 1)_0 \times (\bar{3}, 2, 1, 1)_0 \rightarrow (3, 1, 1, 1)_0, \quad (7.23)$$

with $\Delta C_2 = 4/3$ for $SU(3)_{ETC}$ and the usual $\Delta C_2 = 3/2$ for $SU(2)_{HC}$. This condensation breaks $SU(3)_{ETC}$ to $SU(2)_{TC}$ and preserves the $SU(2)_{HC}$ symmetry. The associated condensate is $\langle \epsilon^{\alpha\beta} \epsilon^{ijk} \zeta_{j,\alpha,R}^T C \zeta_{k,\beta,R} \rangle$. With no loss of generality, we may choose $i = 3$ as breaking direction in $SU(3)_{ETC}$, so that the actual condensate is proportional to

$$\langle \epsilon^{\alpha\beta} \zeta_{5,\alpha,R}^T C \zeta_{6,\beta,R} \rangle. \quad (7.24)$$

We denote the energy scale at which this condensation occurs as Λ_4 . The $\zeta_{j,\alpha,R}$ with $j = 5, 6$ involved in this condensate gain dynamical masses of order Λ_4 , and the five ETC gauge bosons in the coset $SU(3)_{ETC}/SU(2)_{TC}$ gain masses of order $g_{ETC} \Lambda_4 \simeq \Lambda_4$. At this final stage of ETC symmetry breaking, χ_R^4 , ψ_R^{14} , and η_R^{124} decouple from the strong dynamics, since they are singlets under the residual $SU(2)_{TC} \times SU(2)_{HC}$ interaction.

F. Condensation at $\Lambda'_4 < \Lambda_4$

The HC interaction can also produce a condensate involving fermions that are $SU(2)_{TC}$ singlets. Since the formation of this condensate is not aided by the $SU(3)_{ETC}$ interaction, it takes place at a somewhat lower scale than Λ_4 , where α_{HC} has grown to a somewhat larger value. We denote this scale as Λ'_4 . The associated condensate is

$$\langle \epsilon^{\alpha\beta} \zeta_{4,\alpha,R}^T C \omega_{\beta,2,R} \rangle. \quad (7.25)$$

This condensate is invariant under the same strongly coupled symmetry group, $SU_{TC} \times SU(2)_{HC}$, as the condensate (7.24). The $\zeta_{4,\alpha,R}$ and $\omega_{\beta,2,R}$ fermions get dynamical masses of order Λ'_4 due to this condensation. Thus, as the theory evolves below Λ'_4 , all of the HC-nonsinglet fermions have gained masses and have accordingly been integrated out.

G. Theory Below the Energy Scale Λ_4

The theory below Λ_4 is invariant under the strongly coupled groups $SU(2)_{TC} \times SU(2)_{HC}$ and under G_{SM} ,

which is still weakly coupled at this scale. The $SU(2)_{TC}$ acts on the two remaining unbroken ETC indices $j = 5, 6$. This $SU(2)_{TC}$ sector includes the 15 SM-nonsinglet techniquarks and technileptons in Eqs. (2.4) and (2.5), together with three SM-singlet technifermions, χ^j , ψ_R^{1j} , and η_R^{12j} , with $j = 5, 6$, to make a total of 18 chiral doublets, or equivalently, nine Dirac doublets. If the technicolor theory confines and produces the requisite bilinear technifermion condensates, breaking electroweak symmetry, then this may be an acceptable illustrative model of a theory with $N_{gen.} = 4$ SM generations. However, we note the same concern that was mentioned earlier, namely that an $SU(2)_{TC}$ theory with nine Dirac technifermions doublets might evolve into the infrared without producing technifermion condensates and breaking electroweak symmetry. It would be very desirable to use lattice simulations to elucidate the boundary of the chirally symmetric phase of $SU(2)$ as a function of the content of light fermions and to check the Dyson-Schwinger prediction of $N_{f,cr} \simeq 8$ for fermions in the doublet representation. These would constitute a natural extension of the intensive recent lattice work that has been performed for $SU(3)$ [25].

The sequential ETC breakings as discussed above would produce, as desired, a hierarchy of SM fermion masses, with the diagonal elements of the respective mass matrices given by the generic formula (5.14), with $i = 1, 2, 3, 4$, i.e., for the four generations. Assuming that the $SU(2)_{TC}$ sector would confine, it would exhibit strong walking behavior, since the value of N_f is so close to the boundary with the chirally symmetric phase. Hence, the renormalization factor for the fermion bilinears would be $\eta_i \sim \Lambda_4/\Lambda_{TC}$. One could also study the various ETC gauge boson mixings and resultant off-diagonal elements of fermion mass matrices, as well as neutrino masses, especially the requirement of avoiding a fourth light neutrino. However, our results in this section above already demonstrate that one can construct a plausibly tenable model with dynamical EWSB and four SM fermion generations with a corresponding hierarchy of masses.

VIII. DISCUSSION AND CONCLUSIONS

The origin of electroweak symmetry breaking and of the standard-model fermion generations is an outstanding question in particle physics, and it is not yet understood why $N_{gen.} = 3$, rather than some other number. In contrast to the standard model, supersymmetric extensions thereof, and grand unified theories, where one just puts the number $N_{gen.}$ in by hand in a manner that is independent of the gauge group, this number plays a central role in the structure and properties of an extended technicolor model, since it determines what the initial ETC gauge symmetry is, via Eq. (2.3), and how many stages of breaking the ETC symmetry undergoes as it is reduced to the TC subgroup symmetry. In this paper we have taken $N_{gen.}$ as a variable, and have explored

the consequences of varying this number, in the context of models with dynamical EWSB. We have explicitly demonstrated that one can construct TC/ETC models with $N_{gen.} = 1, 2$, and 4, extending the extensive previous work for the physical case of $N_{gen.} = 3$. Our results show that the auxiliary strongly coupled gauge symmetry (hypercolor) is quite useful for obtaining the desired ETC symmetry breaking for these cases $N_{gen.} = 1, 2$, and 4, just as it was for $N_{gen.} = 3$. We have also shown how, for values of $N_{gen.}$ other than 3, one can construct TC/ETC models in which the technicolor theory that results from the sequential ETC symmetry breaking produces the necessary technifermion condensates and plausibly exhibits the desired property of a large but slowly running gauge coupling associated with an approximate infrared-stable fixed point. We have demonstrated that one can build TC/ETC models that can yield generational hierarchies for all of the values of $N_{gen.}$ that we considered. Furthermore, because in each case we were able to obtain a residual technicolor sector that can exhibit walking behavior and hence enhancement of SM fermion masses, the fermions of the highest generation generically have masses that can be comparable in size to the electroweak breaking scale. Stated in other terms, the real-world fact that the top quark has a mass of order the EWSB scale could be shared by fermions of the highest generation in these TC/ETC models with values of $N_{gen.}$ different from 3. It is interesting to compare this result with the situation with the conventional Yukawa mechanism for producing SM fermion masses, where the triviality property of the Yukawa interaction places an upper limit on the Yukawa coupling and hence on the resultant fermion mass. This triviality upper limit on the fermion mass produced by the Yukawa coupling is also comparable to the electroweak symmetry breaking scale, as has been shown by fully nonperturbative, dynamical-fermion lattice simulations [19]. In a theory with strong walking behavior, the effects of SM gauge couplings, which are relatively small perturbations at the TeV scale, could be magnified. However, it is questionable whether the models would produce large intragenerational mass splittings, in particular, between the charge $2/3$ and charge $-1/3$ quarks of a given generation. Our results for $N_{gen.} = 4$ may be useful for those studying the possibility of a real fourth generation. With an illustrative example specifically constructed for the purpose in Sect. IV, we have also illustrated a problem that one can encounter in model-building, in which an excessive number of technifermions can lead to a chirally symmetric evolution of the (asymptotically free) technicolor theory rather than the requisite formation of technifermion condensates at the electroweak scale.

Clearly, TC/ETC theories are subject to a number of severe phenomenological constraints, and one does not yet know if the origin of electroweak symmetry breaking is dynamical, or is due to the vacuum expectation value of a fundamental Higgs field, as hypothesized in the standard model and supersymmetric extensions thereof.

However, we believe that the present study of models with variable $N_{gen.}$ yields useful insights into the role of this number in theories with dynamical electroweak symmetry breaking and can be of value in the continuing quest to understand the origin of standard-model fermion generations. Moreover, since this work involves analyses of patterns of dynamical symmetry breaking of strongly coupled gauge theories, it is also of more abstract field-theoretic interest in its own right. One looks forward eagerly to the elucidation of the physics that is responsible for electroweak symmetry breaking, and an answer to the question of whether it involves strongly or weakly coupled interactions at the TeV scale, that will be forthcoming soon from the Large Hadron Collider.

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IX. APPENDIX

In this appendix we list some formulas that are relevant to our study of the evolution of the TC and HC gauge interactions as functions of energy scale. We consider a (zero-temperature) vectorial $SU(N)$ gauge group with N_f massless Dirac fermions in the fundamental representation. It is assumed that this theory is asymptotically free, i.e., $b_1 > 0$ in Eq. (2.17). Let us define

$$N_{f,IR} = \frac{34N^3}{13N^2 - 3}, \quad (9.1)$$

If $N_f < N_{f,I}$, then $b_2 > 0$, and the only zero of the perturbative two-loop beta function is the zero at the origin (the ultraviolet fixed point of the renormalization group). As N_f increases through the value $N_{f,IR}$, b_2 reverses sign and becomes negative, so that the beta function has a zero away from the origin, at the value

$$\alpha_{IR} = \frac{-4\pi(11N - 2N_f)}{34N^2 - 13NN_f + 3N^{-1}N_f}. \quad (9.2)$$

As the energy scale μ decreases from large values, α increases toward the value α_{IR} , which is thus an infrared fixed point of the renormalization group. It is (i) an exact IR fixed point if there is no change in the massless particle content as α increases toward α_{IR} from below, or alternatively (ii) an approximate IR fixed point if, as α increases toward α_{IR} , it exceeds a critical value α_{cr} for spontaneous chiral symmetry breaking via the formation of bilinear condensates of the fermions at some scale $\mu = \Lambda_{cr}$. In the latter case, these fermions gain dynamical masses and are integrated out in the effective low-energy theory that is applicable for $\mu < \Lambda_{cr}$; as a consequence, the massless particle content of the theory changes and it evolves further into the infrared in a manner governed by a different set of coefficients in the beta function.

As N_f increases above $N_{f,IR}$, the value of α_{IR} decreases, and as N_f increases through a critical value $N_{f,cr}$, α_{IR} decreases below the minimum value, α_{cr} , for condensate formation. For $N_{f,cr} < N_f < (11/2)N$, the theory is therefore in a chirally symmetric phase. In accord with physical arguments connecting confinement and spontaneous chiral symmetry breaking [43], this is often inferred to be a conformal, non-Abelian Coulomb phase. This inference is clearly valid in the limit where N_f approaches $(11/2)N$ from below, so that b_1 and α_{IR} become very small, and the gauge interaction become very weak. The inference assumes that in this phase without any fermion condensates, where the fermions do not pick up any dynamical masses, there are also no glueballs; i.e., the glueballs of the confined phase have become unbound, producing free massless gluons. The value of $N_{f,cr}$ is determined by setting $\alpha_{IR} = \alpha_{cr}$, yielding the result [23]

$$N_{f,cr} = \frac{2N(50N^2 - 33)}{5(5N^2 - 3)}. \quad (9.3)$$

For $N = 2$, this gives $N_{f,cr} \simeq 8$ (and for $N = 3$ it gives $N_{f,cr} \simeq 12$). This is the basis for the statement that a one-family technicolor theory, which has $2(N_c + 1) = 8$ Dirac technifermions, plausibly exhibits walking behavior. Clearly, Eq. (2.22) and the resultant Eq. (9.3) are only rough estimates, in view of the strongly coupled nature of the physics and the fact that this approach neglects nonperturbative effects, such as instantons, which enhance chiral symmetry breaking [44]. Moreover, the Dyson-Schwinger equation does not incorporate confinement, and the condition in Eq. (2.22) is obtained by doing a loop integration over all Euclidean loop momenta, but in fact the integration range is reduced, since a particle confined within a size $r \sim 1/\Lambda$ has a maximum wavelength and equivalently, a minimum boundstate momentum of order Λ , the confinement energy scale [38]. Fortunately, these two omissions (instantons and reduction of the integration range in the Dyson-Schwinger integral) affect the prediction for α_{cr} in opposite ways, so that the omission of both of them may not be too serious. As noted in the text, a continuum study of corrections to the one-gluon exchange approximation in solving the Dyson-Schwinger equation found it to be reasonably accurate [41]. More recently, in the case of $SU(3)$, lattice studies yield results that are broadly consistent with the predictions of the earlier Dyson-Schwinger analysis [25]. In this context, one should note that the study of the Dyson-Schwinger equation for the fermion propagator only gives information about chiral symmetry breaking; this equation does not directly contain information about confinement. In principle, if appropriate conditions were satisfied [45] (which are necessary but not sufficient conditions), one could have a confined phase without spontaneous chiral symmetry breaking. However, this possibility is not relevant for our present analysis, since we require that there be S χ SB in the ETC and HC sectors to produce the sequential ETC symmetry breaking, and

in the TC sector to produce the electroweak symmetry breaking.

For our analyses of the successive stages of ETC symmetry breaking we will apply this sort of method for a chiral gauge theory with a general set of fermion representations. In this case, in terms of the chiral fermion representations R , the first two coefficients of the beta function are [46]

$$b_1 = \frac{1}{3} \left[11C_2(G) - 2 \sum_R T(R) N_{f,R} \right] \quad (9.4)$$

and [47]

$$b_2 = \frac{1}{3} \left[34C_2(G)^2 - \sum_R [10C_2(G) + 6C_2(R)] T(R) N_{f,R} \right]. \quad (9.5)$$

Higher coefficients are scheme-dependent, and these first two, which are scheme-independent, will suffice for our purposes. For a vectorial theory, the left- and right-handed chiral fermions of representation R are combined into a single Dirac fermion in this representation. It is not necessary that the enveloping ETC group or the in-

termediate subgroups above the level of $SU(2)_{TC}$ exhibit walking behavior; our only constraints for these groups are (i) that they be asymptotically free and (ii) that their fermion content be such that as they evolve into the infrared, they spontaneously break chiral symmetry via formation of fermion condensates instead of evolving into a non-Abelian Coulomb phase. This does not require a perturbative infrared zero of the beta function.

In particular, given that the $SU(2)_{HC}$ gauge interaction has an even number, $N_{f,1/2}$, of chiral fermions transforming as doublet representations, as it must to avoid a global anomaly, it can always be rewritten as a vectorial gauge theory with $N_{f,D,1/2} = N_{f,1/2}/2$ Dirac doublets. Hence for this HC theory we have

$$b_1 = \frac{1}{3} (22 - 2N_{f,D,1/2}) \quad (9.6)$$

and

$$b_2 = \frac{1}{3} \left[136 - \frac{49N_{f,D,1/2}}{2} \right]. \quad (9.7)$$

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- [1] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).
 - [2] S. Dimopoulos and L. Susskind, Nucl. Phys. B **155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. B **90**, 125 (1980).
 - [3] R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973); J. Cornwall and R. Norton, Phys. Rev. D **8**, 3338 (1973); S. Weinberg, Phys. Rev. D **13**, 974 (1976); S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. B **169**, 373 (1980).
 - [4] R. Shrock, in *Proc. Internat. Workshop on the Origin of Mass and Strongly Coupled Gauge Theories - SCGT06*, eds. M. Harada, M. Tanabashi, and K. Yamawaki (World Scientific, Singapore, 2008), p. 227 (hep-ph/0703050).
 - [5] R. S. Chivukula, M. Narain, and J. Womersley, Phys. Lett. B **667**, 1258 (2008) and <http://pdg.lbl.gov>.
 - [6] See talks in *Workshop on Dynamical Electroweak Symmetry Breaking*, Southern Denmark University, 2008, <http://hep.sdu.dk/dewsb>.
 - [7] F. Sannino, arXiv:0911.0931.
 - [8] More precisely, for SM generations with associated neutrinos that are light compared with $m_Z/2$, LEP has determined that $N_\nu = N_{gen.} = 3$ from measurements of the invisible width of the Z . This result is also consistent with constraints on light neutrinos from primordial nucleosynthesis. Constraints on a possible fourth generation are noted in the text.
 - [9] T. Appelquist and J. Terning, Phys. Rev. D **50**, 2116 (1994).
 - [10] T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002).
 - [11] T. Appelquist and R. Shrock, Phys. Rev. Lett. **90**, 201801 (2003).
 - [12] T. Appelquist, M. Piai, and R. Shrock, Phys. Rev. D **69**, 015002 (2004).
 - [13] T. Appelquist, M. Piai, and R. Shrock, Phys. Lett. B **593**, 175 (2004); Phys. Lett. B **595**, 442 (2004).
 - [14] T. Appelquist, N. D. Christensen, M. Piai, and R. Shrock, Phys. Rev. D **70**, 093010 (2004).
 - [15] P. H. Frampton, P. Q. Hung, and M. Sher, Phys. Rep. **330**, 263 (2000).
 - [16] J. Erler and P. Langacker, in *Review of Particle Properties*, Phys. Lett. B **667**, 125 (2008) and [20].
 - [17] B. Holdom, W. S. Hou, T. Hurth, M. Mangano, S. Sultanosoy, and G. Ünsel, PMC Physics A **3**, 4 (2009) (arXiv:0904.4698); A. Soni, arXiv:0907.2057.
 - [18] Some recent papers include V. A. Novikov, A. N. Rozanov, and M. I. Vysotsky, Yad. Fiz., to appear (arXiv:0904.4570); M. Hashimoto and V. A. Miransky, Phys. Rev. D **80**, 013004 (2009); E. De Pree, G. Marshall, and M. Sher, Phys. Rev. D **80**, 037301 (2009); P. Q. Hung and C. Xiong, arXiv:0911.3890; arXiv:0911.3892.
 - [19] I-H. Lee, J. Shigemitsu, and R. Shrock, Nucl. Phys. B **330**, 225 (1990); Nucl. Phys. B **334**, 265 (1990); R. Shrock, in M. Creutz, ed., *Quantum Fields on the Lattice* (World Scientific, Singapore, 1992), p. 150.
 - [20] See <http://pdg.lbl.gov>.
 - [21] B. Holdom, Phys. Lett. B **150**, 301 (1985); B. Holdom, Phys. Rev. Lett. **60**, 1233 (1988); a study for a UV-stable fixed point is K. Yamawaki, M. Bando, and K. Matsumoto, Phys. Rev. Lett. **56**, 1335 (1986).
 - [22] T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); Phys. Rev. D **36**, 568 (1987).
 - [23] T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996); Phys. Rev. Lett. **79**, 2767 (1997).
 - [24] R. S. Chivukula, Phys. Rev. D **55**, 5238 (1997).

- [25] T. Appelquist, G. Fleming, and E. Neil, Phys. Rev. Lett. **100**, 171607 (2008); Phys. Rev. D **79**, 076010 (2009); A. Deuzeman, M. P. Lombardo, and E. Pallante, Phys. Lett. B **670**, 41 (2008); arXiv:0810.1719, arXiv:0904.4662; X.-Y. Jin and R. Mawhinney, PoS Lattice-2008:059 (2008), arXiv:0812.0413; Z. Fodor, K. Holland, J. Kuti, D. Nogradi, and C. Schroeder, arXiv:0809.4890, arXiv:0907.4562, arXiv:0908.2466.
- [26] K. Lane and E. Eichten, Phys. Lett. B **222**, 274 (1989).
- [27] F. Sannino and K. Tuominen, Phys. Rev. D **71**, 051901 (2005); N. D. Christensen and R. Shrock, Phys. Lett. B **632**, 92 (2006); D. D. Dietrich and F. Sannino, Phys. Rev. D **75**, 085018 (2007); R. Foadi, M. T. Frandsen, T. A. Ryttov, and F. Sannino, Phys. Rev. D **76**, 055005 (2007); S. B. Gudnason, T. A. Ryttov, and F. Sannino, Phys. Rev. D **76**, 015005 (2007); T. A. Ryttov and F. Sannino, Phys. Rev. D **76**, 105004 (2007).
- [28] M. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992); R. Cahn and M. Suzuki, Phys. Rev. D **44**, 3641 (1991); T. Appelquist and F. Sannino, Phys. Rev. D **59**, 067702 (1999); S. Ignjatovic, L. C. R. Wijewardhana, and T. Takeuchi, Phys. Rev. D **61**, 056006 (2000).
- [29] N. D. Christensen and R. Shrock, Phys. Rev. Lett. **94**, 241801 (2005).
- [30] M. Harada, M. Kurachi, and K. Yamawaki, Prog. Theor. Phys. **115**, 765 (2006); M. Kurachi and R. Shrock, Phys. Rev. D **74**, 056003 (2006); M. Kurachi, R. Shrock, and K. Yamawaki, Phys. Rev. D **76**, 035003 (2007); K. Agashe, C. Csáki, M. Reece, and C. Grosjean, JHEP **12**, 003 (2007); J. Hirn, A. Martin, and V. Sanz, Phys. Rev. D **78**, 075026 (2008); K. Lane and A. Martin, Phys. Rev. D **80**, 115001 (2009).
- [31] See, e.g., Y. Tosa and R. E. Marshak, Phys. Rev. D **27**, 616 (1983) and references therein.
- [32] See, e.g., M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge Univ. Press, Cambridge, 1986), vol. 2, p. 408.
- [33] E. Farhi and L. Susskind, Phys. Rev. D **20**, 3404 (1979); N. D. Christensen and R. Shrock, Phys. Rev. D **72**, 035013 (2005); N. Chen and R. Shrock, Phys. Rev. D **78**, 035002 (2008).
- [34] N. D. Christensen and R. Shrock, Phys. Rev. D **74**, 015004 (2006).
- [35] C. Hill, Phys. Lett. B **345**, 483 (1995); R. S. Chivukula, B. Dobrescu, and J. Terning, Phys. Lett. B **353**, 289 (1995); K. Lane and E. Eichten, Phys. Lett. B **352**, 382 (1995); K. Lane, Phys. Lett. B **433**, 96 (1998); C. Hill and E. Simmons, Phys. Rep. **381**, 235 (2003); A. Martin and K. Lane, Phys. Rev. D **71**, 015011 (2005).
- [36] For our present purposes, it will be sufficient to restrict our consideration to models in which the hypercolor symmetry is never broken. We note, however, that in one of the breaking sequences considered earlier, denoted G_b in Ref. [10] and sequence S_2 in Ref. [12], $SU(2)_{HC}$ is broken to the (non-asymptotically free) $U(1)_{HC}$.
- [37] S. J. Brodsky and G. F. de T ramond, in, *Proc. Internat. Workshop on the Origin of Mass and Strongly Coupled Gauge Theories - SCGT06*, eds. M. Harada, M. Tanabashi, and K. Yamawaki (World Scientific, Singapore, 2008), p. 31, and in further talks posted at <http://www.slac.stanford.edu/grp/th/Brodsky>.
- [38] S. J. Brodsky and R. Shrock, Phys. Lett. B **666**, 95 (2008).
- [39] M. Creutz, unpublished and private communications.
- [40] The quadratic Casimir invariant for a group G is given by $\sum_{j=1}^{dim(R)} [D_R(T_a)]_{ij} [D_R(T_a)]_{jk} = C_2(R) \delta_{ik}$, where a, b are group indices, T_a are the generators of the associated Lie algebra, and $D_R(T_a)$ are the matrix representations of the T_a in the representation R .
- [41] T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61**, 1553 (1988).
- [42] C. Quigg and R. Shrock, Phys. Rev. D **79**, 096002 (2009).
- [43] A. Casher, Phys. Lett. B **83**, 395 (1979).
- [44] T. Appelquist and S. Selipsky, Phys. Lett. B **400**, 364 (1997); M. Velkovsky and E. Shuryak, Phys. Lett. B **437**, 398 (1998).
- [45] G. 't Hooft, in *Recent Developments in Gauge Theories* (1979 Carg se Summer Institute) (Plenum, New York, 1980), 135.
- [46] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); Phys. Rev. D **8**, 3633 (1973); H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973); G. 't Hooft, unpublished.
- [47] W. E. Caswell, Phys. Rev. Lett. **33**, 244 (1974); D. R. T. Jones, Nucl. Phys. B **75**, 531 (1974).